Unit 3, Lesson 1
All Things Being Equal

Learning Focus

Solve quadratic equations graphically and algebraically.

Make connections between solving quadratic equations and graphing quadratic functions.

How can we use graphs to solve quadratic equations?

Open Up the Math: Launch, Explore, Discuss

As we have learned, quadratic functions can be very useful models for a lot of real situations. They are part of understanding the motion of objects, business and economic models, and many other applications. Using quadratic functions often requires solving a quadratic equation. This can be a pretty straightforward process, or it can be a little complicated. The good news is that we know a lot about quadratic functions that we can apply to solving equations. Let's get started.

1. If you were given the quadratic equation, $x^2 = 9$, how would you solve it? What solution(s) would you get?

2. Graph the function: $f(x) = x^2$.

    a. For what values of x does $f(x) = 9$? How did you find the values? How do they compare to the solutions of the equation?

3. Given the equation $(x-1)^2 = 4$:

    a. Solve the equation algebraically.

    b. Graph the function $f(x) = (x-1)^2$ and mark the points where $f(x) = 4$.

4. Given the equation: $-x^2 - 6x - 7 = 1$

    a. Graph the function $f(x) = -x^2 - 6x - 7$ and find where $f(x) = 1$. (Just to be efficient, use technology and record your graph here with the solution points marked.)

    b. Solve the equation algebraically.

Don't feel bad if you couldn't solve the equation algebraically. It's one of those complicated equations that can't be solved directly with inverse operations. Knowing that the graph of an equation is the set of all of its solutions, we can solve any quadratic equation graphically using technology. Now we're going to use our understanding of the graph of a quadratic function and its symmetry to develop other algebraic techniques for solving quadratic equations.

Let's start in a familiar place.

5. If you're given the function $f(x) = (x-4)(x+3)$ and asked to find the x-intercepts, what quadratic equation would you be solving?

6. How could you use this kind of thinking to solve: $x^2 - x - 12 = 0$?

7. Now you can try a few. Solve each quadratic equation by factoring.

    a. $x^2 - 9x + 20 = 0$

    b. $3x^2 - 14x - 5 = 0$

    c. $2x^2 + 8x - 42 = 0$

When we were graphing, we saw that some functions are easier to factor, and others are easier to complete the square. Let's start with one where the work has been done for us so that we can start seeing some relationships.

8. Consider the function: $f(x) = (x-1)^2 - 4$ and the equation $0 = (x-1)^2 - 4$.

    a. Graph the function:

    b. Line of symmetry:

    c. Vertex:

    d. What are the x-intercepts of $f(x)$?

    e. How far are the x-intercepts from the line of symmetry? (Think of the distance left and right.)

    f. Use inverse operations to solve the equation $0 = (x-1)^2 - 4$.

    g. How does solving the equation relate to the graph and the x-intercepts?

Don't be mad, but that equation would have factored easily if it were given in standard form. That's why the solutions were integers. Not all equations factor easily, and the relationship that we are beginning to see can be very helpful.

Let's look at a function and equation that do not factor easily from standard form, and see how vertex form can help us solve the equation or find the x-intercepts.

9. Start with the function $f(x) = x^2 - 6x + 4$.

    a. Graph the function by putting it in vertex form.

    b. Vertex:

    c. Line of symmetry:

    d. What do you estimate the x-intercepts to be?

    e. What do you estimate the distance from each x-intercept to the line of symmetry to be?

    f. Starting with $f(x)$ in vertex form, write and solve the equation $f(x) = 0$ using inverse operations.

    g. How do your solutions compare with your estimates of the x-intercepts and the distance from the line of symmetry and the x-intercepts?

10. Given: $x^2 - 6x + 7 = 0$

    a. Solve the equation by completing the square and using inverse operations.

    b. Let $f(x) = x^2 - 6x + 7$. What is the line of symmetry?

    c. What is the distance from line of symmetry to an x-intercept?

11. Given: $2x^2 - 8x + 5 = 0$

    a. Solve the equation by completing the square and using inverse operations.

    b. Let $f(x) = 2x^2 - 8x + 5$. What is the line of symmetry?

    c. What is the distance from line of symmetry to an x-intercept?

Ready for More?

Find two methods for solving this equation graphically:

$3x^2 + 3x - 5 = -x + 7$

Takeaways

Solving quadratic equations using inverse operations:

Example	Procedure
$(x+2)^2 = 16$	Given

Solving quadratic equations by graphing:

Example	Procedure
$(x+2)^2 = 16$	Given

Solving quadratic equations by factoring:

Example	Procedure
$x^2 + 4x - 12 = 0$	Given _____

Solving quadratic equations by completing the square:

Example	Procedure
$x^2 + 4x - 12 = 0$	Given _____

Vocabulary

- **zero property of multiplication (also called the zero product property)**

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we learned methods for solving quadratic equations. Some quadratic equations can be solved using inverse operations and taking the square root of both sides of the equations. Some quadratic equations can be solved by factoring and using the zero product property. Some quadratic equations can be solved by completing the square and then using inverse operations. Quadratic equations that have real solutions can also be solved by graphing, and each of these algebraic methods has connections to graphing.

Unit 3, Lesson 2
Formula for Success

Jump Start

1. Solve the equation by completing the square and using inverse operations:

   $3x^2 - 6x - 4 = 0$

2. Given $f(x) = 3x^2 - 6x - 4$, use your solutions from problem 1 to find:

   a. Line of symmetry:

   b. The distance from the line of symmetry to the x-intercepts:

Learning Focus

Understand and use a formula for solving quadratic equations.

Is there any method that works for solving all quadratic equations?

Open Up the Math: Launch, Explore, Discuss

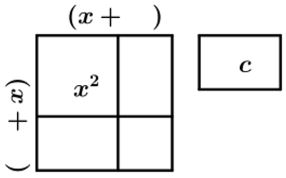
Solving an equation by completing the square seems like it goes exactly the same way every time. Let's try to generalize this process by using: $f(x) = ax^2 + bx + c$ to represent any quadratic function that has x-intercepts and $ax^2 + bx + c = 0$.

Start the process the usual way by putting the equation into vertex form. It's a little tricky, but just do the same thing with a, b, and c as what you did in the last problem with the numbers.

Here are some steps to think about using a diagram:

1. How many big blocks do you have?

2. Imagine one of the big blocks and the extra smaller blocks. What goes into each of the sections? You've been given a start below. Try filling in the other sections.



Now that you have filled in the small square, what expression do you need to subtract from c? Don't forget that you have a big blocks and the diagram shows just one of them. Complete the diagram including the lengths of the sides.

3.

   a. What is the equivalent expression for $f(x) = ax^2 + bx + c$ in vertex form?

   b. What is the line of symmetry of the parabola?

   c. What is the vertex of the parabola?

4. Write and solve the equation for $f(x) = 0$ just using inverse operations and some fancy algebra.

Solve each equation using the quadratic formula.

5. $x^2 + 5x + 1 = 0$
6. $x^2 - 5x - 5 = 0$
7. $x^2 - 6x - 3 = 0$
8. $2x^2 + 8x + 5 = 0$
9. Now that you've tried using the quadratic formula, here are a few questions to consider. Record your answers below.
    a. Why do the solutions from the quadratic formula contain a "$\pm$"?
    b. Is it possible to have only one solution to a quadratic equation? What would that mean about the associated function?
    c. What does it mean if the number inside the root of a solution turns out to be negative?

Ready for More?

Solve this equation using the quadratic formula $2x^2 + 6x + 5 = 0$.

How do you interpret the solutions?

Examine the graph of the related function after finding the solutions that are undefined as real numbers.

Takeaways

The quadratic formula:

Vocabulary

- **quadratic formula**

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we found a formula by completing the square for solving quadratic equations. We used the quadratic formula to find exact and approximate solutions to quadratic equations and connected those solutions to the graphs of quadratic functions. We learned to rewrite exact solutions so that fractions do not contain common factors in the numerator and denominator and the square roots do not contain factors that are perfect squares.

Unit 3, Lesson 3

Comparison Shopping

Learning Focus

Solve quadratic equations efficiently and accurately.

Solve systems of quadratic and linear equations.

How can you determine the most efficient strategy for solving any particular quadratic equation?

Open Up the Math: Launch, Explore, Discuss

In this lesson, you will be comparing methods for solving quadratic equations, looking for which method works best for a given equation. The methods you will compare along the way are:

- Solving by factoring
- Solving by completing the square and using inverse operations
- Solving by using the quadratic formula
- Solving by graphing

You should also be thinking about polishing your skills in using each of these methods by checking your work and watching for repeated errors, like mixing up the signs or making mistakes in dividing common factors out of fractions. In other words, don't just work the problems to get them done; work the problems so you're getting better!

1. Given: $x^2 - 2x - 15 = 0$
    a. Solve by completing the square:
    b. Solve by factoring:
2. Given: $2x^2 + 3x + 1 = 0$
    a. Solve by factoring:
    b. Solve using the quadratic formula:
3. Given: $x^2 + 4x - 8 = 0$
    a. Solve by completing the square:
    b. Solve using the quadratic formula:
    c. Can this equation be solved by factoring? How can you tell?

4. Given the system of equations:

$$\begin{aligned} y_1 &= x^2 - 4x + 1 \\ y_2 &= x - 3 \end{aligned}$$

   a. Solve by graphing:

   b. Solve algebraically:

Choose any algebraic method, and solve each equation or system of equations. Check your solutions by graphing. Be prepared to share the reason you selected the method you used to solve the equation(s).

5. $4x^2 - 81 = 0$

6. $2x^2 + 4x = 7$

7. $(x + 8)(x + 6) = 288$

8. $$\begin{aligned} y_1 &= x^2 + x - 2 \\ y_2 &= -x + 1 \end{aligned}$$

Ready for More?

You've solved some quadratic equations, and now you have a chance to write your own. Try these challenges.

1. Write a quadratic equation that has only one solution.

2. Write a quadratic equation that has two rational solutions that are not integers.

3. Write a quadratic equation that has no solutions.

Takeaways

Efficient methods for solving quadratic equations in the form $ax^2 + bx + c = 0$.

Factoring is an efficient strategy when: _____

Completing the square is an efficient strategy when: _____

Quadratic formula is an efficient strategy when: _____

Graphing is an efficient strategy when: _____

Lesson Summary

In this lesson, we compared methods for solving quadratic equations. We found that some equations lend themselves to one method, and other equations are more efficiently solved with other methods. Using technology to graph is always a useful way to check solutions.

Unit 3, Lesson 4

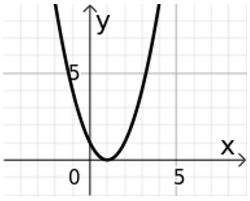
To Be Determined

Jump Start

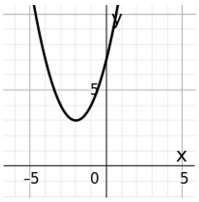
Which One Doesn't Belong?

Examine each of the quadratic functions given below and determine which one is not like the others.

A. $f(x) = (x - 3)(x + 2)$

B. 

C. $f(x) = x^2 + 3x - 7$

D. 

Reason:

Learning Focus

Write quadratic functions in vertex, factored, and standard form.

Find roots of a quadratic function.

Use the roots of a quadratic function to write the function in factored form.

Can every quadratic function be written in all three of the forms we have studied: standard, vertex, and factored form?

How do solutions of a quadratic relate to the factors of a quadratic function?

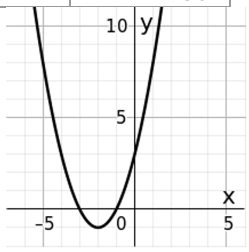
Do all quadratic functions have two roots?

Open Up the Math: Launch, Explore, Discuss

Israel and Miriam are working together on a homework assignment. They need to write the equations of quadratic functions from the information given in a table or a graph. At first, this work seemed really easy. However, as they continued to work on the assignment, the algebra got more challenging and raised some interesting questions that they can't wait to ask their teacher. Work through the following problems from Israel and Miriam's homework. Use the information in the table or the graph to write the equation of the quadratic function in all three forms. You may start with any form you choose, but you need to find all three equivalent forms.

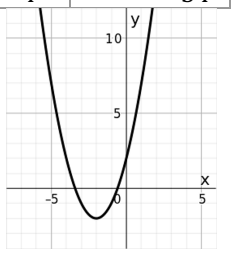
1.

x	y
-5	8
-4	3
-3	0
-2	-1
-1	0
0	3
1	8
2	15
3	24
4	35



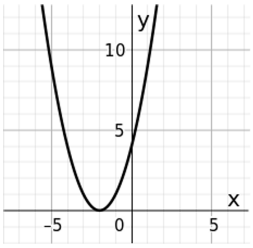
2.

x	y
-5	7
-4	2
-3	-1
-2	-2
-1	-1
0	2
1	7
2	14
3	23
4	34



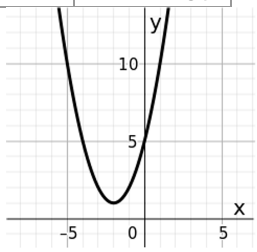
3.

x	y
-5	9
-4	4
-3	1
-2	0
-1	1
0	4
1	9
2	16
3	25
4	36



4.

x	y
-5	10
-4	5
-3	2
-2	1
-1	2
0	5
1	10
2	17
3	26
4	37



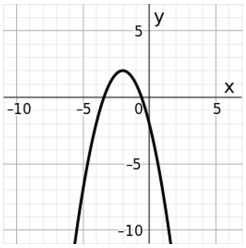
5. Israel was concerned that his factored form for the function in problem 4 didn't look quite right. Miriam suggested that he test it out by substituting in some values for x to see if he gets the same points as those in the table. Test your factored form. Do you get the same values as those in the table?

6. Why might Israel be concerned about writing the factored form of the function in problem 4?

Here are some more examples from Israel and Miriam's homework.

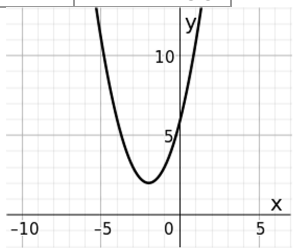
7.

x	y
-5	-7
-4	-2
-3	1
-2	2
-1	1
0	-2
1	-7
2	-14
3	-23
4	-34



8.

x	y
-5	11
-4	6
-3	3
-2	2
-1	3
0	6
1	11
2	18
3	27
4	38



9. Miriam notices that the graphs of the functions in problems 7 and 8 have the same vertex point. Israel notices that the graphs of the functions in problems 2 and 7 are mirror images across the x-axis. What do you notice about the roots of the functions in problems 7, 8, and 2?

The Fundamental Theorem of Algebra

A polynomial function is a function of the form:

$y = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots a_{n-3}x^3 + a_{n-2}x^2 + a_{n-1}x + a_n$

where all of the exponents are positive integers and all of the coefficients $a_0 \ldots a_n$ are constants.

As the theory of finding roots of polynomial functions evolved, a 17th century mathematician, Girard (1595-1632) made the following claim which has come to be known as the Fundamental Theorem of Algebra: *An n^{th} degree polynomial function has n roots*.

10. In the next unit you will study polynomial functions that contain higher-ordered terms such as x^3 or x^5. Based on your work in this task, do you believe this theorem holds for quadratic functions? That is, do all functions of the form $y = ax^2 + bx + c$ always have two roots? Examine the graphs of each of the quadratic functions you have written equations for in this task. Do they all have two roots? Why or why not?

Ready for More?

Square roots can often be written in equivalent forms by looking for perfect square factors that can be removed from the radicand. For example, $\sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$. We can verify that the number $2\sqrt{2}$ is equivalent to $\sqrt{8}$ by multiplying it by itself and using the commutative property of multiplication to rearrange the factors: $2\sqrt{2} \cdot 2\sqrt{2} = 2 \cdot 2 \cdot \sqrt{2} \cdot \sqrt{2} = 2 \cdot 2 \cdot 2 = 8$. If we allow, at least temporarily, $\sqrt{-1}$ to be a legitimate factor, show how you might represent $\sqrt{-8}$ in an equivalent form and then verify that your new representation for $\sqrt{-8}$ works by multiplying it by itself. Based on this work, rewrite the following radical expressions:

1. $\sqrt{-75}$
2. $\sqrt{-36}$
3. $\sqrt{-32}$
4. $\sqrt{-48}$

Takeaways

Observations about the x-intercepts and roots of a quadratic function:

Vocabulary

- **Fundamental Theorem of Algebra**
- roots: real and imaginary
- x-intercept
- zeros, roots, solutions

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we examined solutions to quadratic equations and connected them with the graph of the function. Solutions for a quadratic equation can be used to write the function in factored form in a process that is the reverse of solving an equation by factoring. We found that when the graph of the quadratic function did not cross the x-axis, the quadratic formula gave solutions that included the square root of a negative number. Although there is not a real number that can be squared to get a negative number, these expressions seemed to work like square roots. Since the Fundamental Theorem of Algebra predicts that all quadratic functions will have two roots, the nature of solutions that involve the square root of a negative number still needs to be resolved.

Unit 3, Lesson 5
I May Be Irrational, But You're Imaginary

Learning Focus

Relate irrational numbers to physical quantities such as the hypotenuse of a right triangle.

Understand expressions that contain negative numbers inside a square root, like $\sqrt{-1}$.

Add, subtract, and multiply complex numbers.

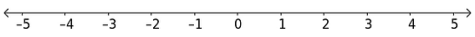
Can numbers like $\sqrt{-1}$ be rewritten?

What types of numbers make up the number system?

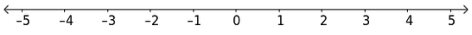
Open Up the Math: Launch, Explore, Discuss

Part 1: Irrational numbers

1. Verify that $4\left(x-\frac{5}{2}\right)\left(x+\frac{3}{2}\right)=0$ and $4x^2-4x-15=0$ are equivalent equations (show your work), and plot the solutions to the quadratic equations on the following number line:

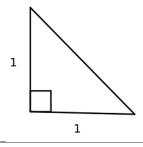


2. Verify that $\left(x-2+\sqrt{2}\right)\left(x-2-\sqrt{2}\right)=0$ and $x^2-4x+2=0$ are equivalent equations (show your work), and plot the solutions to the quadratic equations on the following number line:



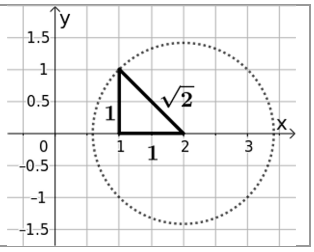
You may have found it difficult to locate the exact points on the number line that represent the two solutions to the second pair of quadratic equations given above. The following diagrams might help.

3.

Find the perimeter of this isosceles triangle. Express your answer with any like terms combined.	

We might approximate the perimeter of this triangle with a decimal number, but the exact perimeter is $2+\sqrt{2}$. Note that this notation represents a single number—the distance around the perimeter of the triangle—even though it is written as the sum of two terms.

4.

Explain how you could use this diagram to locate the two solutions to the quadratic equations given in problem 2: $2+\sqrt{2}$ and $2-\sqrt{2}$.	

5. Are the numbers we have located on the number line in this way rational numbers or irrational numbers? Explain your answer.

Both sets of quadratic equations given in problems 1 and 2 have solutions that can be plotted on a number line. The solutions to the first set of quadratic equations are rational numbers. The solutions to the second set of quadratic equations are irrational numbers.

Big Idea #1: The set of numbers that contains all of the **rational numbers** and all of the **irrational numbers** is called the set of **real numbers**. The location of all points on a number line can be represented by real numbers.

Pause and Reflect:

Part 2: Imaginary and Complex Numbers

In the previous lesson, To Be Determined, you found that the quadratic formula gives the solutions to the quadratic equation $x^2 + 4x + 5 = 0$ as $-2 + \sqrt{-1}$ and $-2 - \sqrt{-1}$. Because the square root of a negative number has no defined value as either a rational or an irrational number, Euler proposed that a new number $i = \sqrt{-1}$ be included in what came to be known as the complex number system.

6. Based on Euler's definition of i, what would the value of i^2 be?

With the introduction of the number i, the square root of *any* negative number can be represented. For example, $\sqrt{-2} = \sqrt{2} \cdot \sqrt{-1} = \sqrt{2} \cdot i$ and $\sqrt{-9} = \sqrt{9} \cdot \sqrt{-1} = 3i$.

7. Find the values of the following expressions. Show the details of your work.

   a. $\left(\sqrt{2} \cdot i\right)^2$

   b. $3i \cdot 3i$

Big Idea #2: Numbers like $3i$ and $\sqrt{2} \cdot i$ are called pure *imaginary numbers*. Numbers like $-2 - i$ and $-2 + i$ that include a real term and an imaginary term are called *complex numbers*.

Big Idea #3: Complex numbers are not real numbers—they do not lie on the real number line that includes all of the rational and irrational numbers; also note that the real numbers are a subset of the complex numbers since a real number results when the imaginary part of $a + bi$ is 0, that is, $a + 0i$.

Operations with complex numbers follow the same properties as real numbers, but there are a few things to pay attention to, like when adding or subtracting two complex numbers, the real parts of the numbers are like terms and the imaginary part of the numbers are like terms.

For example: $(3 + 4i) + (5 - 2i) = 8 + 2i$. If you want to line them up vertically like a polynomial, it will keep the like terms organized. When complex numbers are subtracted, it helps to change the signs of the number being subtracted and add just like we did with polynomials.

For example: $(7 + 2i) - (3 - 5i)$ can be rewritten as $(7 + 2i) + (-3 + 5i) = 4 + 7i$.

Now try a few yourself:

8.

a. $(13 + 2i) + (-12 - 5i)$

b. $(-7 + 12i) + (3 - 6i)$

c. $(8 - i) - (7 - 3i)$

d. $(-9 + 4i) - (9 - 4i)$

Multiplying complex numbers is the same as multiplying irrational numbers like $(2 + \sqrt{3})(1 - \sqrt{5})$ or binomials like $(x + 5)(x - 2)$ with one added step. Anytime an i^2 shows up, it needs to be written as -1 and then the expression can be rewritten.

For instance, let's multiply $(4 + 2i)(3 - 5i)$ using the open area model method. Adding like terms gives us: $12 - 14i - 10i^2$. Now substitute $i^2 = -1$ and rewrite the expression by combining like terms: $12 - 14i - 10(-1) = 12 - 14i + 10 = 22 - 14i$

	4	$+2i$
3	12	$6i$
$-5i$	$-20i$	$-10i^2$

Give it a try, using either the open area model method or distributing terms horizontally.

9.

a. $(3 + 2i)(5 + 7i)$

b. $(10 - 3i)(4 - 6i)$

These two complex numbers are called conjugates because they are the same except the opposite sign on the second term. Check out what happens here:

10.

a. $(4 + 2i)(4 - 2i)$

b. $(3 + 5i)(3 - 5i)$

c. What do you notice about the answers? Why is this result occurring?

Now let's go back and think about complex solutions to quadratic equations.

11. Given the function $f(x) = x^2 + 4x + 5$, find x such that $f(x) = 0$.

12. Check one of your solutions, showing the details of your work.

13. Are the solutions to the equation $x^2 + 4x + 5 = 0$ the x-intercepts of $f(x)$? Explain your answer.

14. Find the solutions to the equation $x^2 - 3x + 4 = 0$

Now that we have established complex numbers as functional, useful numbers, let's think again about the Fundamental Theorem of Algebra.

The Fundamental Theorem of Algebra, Revisited

Remember the following information given in the previous task:

A polynomial function is a function of the form: $y = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots a_{n-3}x^3 + a_{n-2}x^2 + a_{n-1}x + a_n$

where all of the exponents are positive integers and all of the coefficients $a_0 \ldots a_n$ are constants.

As the theory of finding roots of polynomial functions evolved, a 17th century mathematician, Girard (1595-1632) made the following claim which has come to be known as the Fundamental Theorem of Algebra: *An n^{th} degree polynomial function has n roots.*

15. Based on your work in this task, do you believe this theorem holds for quadratic functions? That is, do all functions of the form $y = ax^2 + bx + c$ always have two roots?

Ready for More?

The solutions to the equation $x^2 + 4x + 5 = 0$ can be written as $-2 + i$ and $-2 - i$, and the factored form of $x^2 + 4x + 5$ can be written as $(x + 2 - i)(x + 2 + i)$.

Verify that $x^2 + 4x + 5$ and $(x + 2 - i)(x + 2 + i)$ are equivalent by expanding and combining like terms to get the standard form. Show the details of your work.

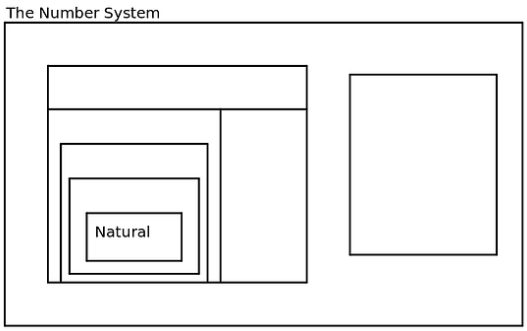
Takeaways

x-intercepts: _____

Roots: _____

If the graph of a quadratic function intersects the x-axis, _____

If the graph of a quadratic function does not intersect the x-axis, _____



Vocabulary

- **complex conjugates**
- **complex number**
- **imaginary number**

Bold terms are new in this lesson.

Lesson Summary

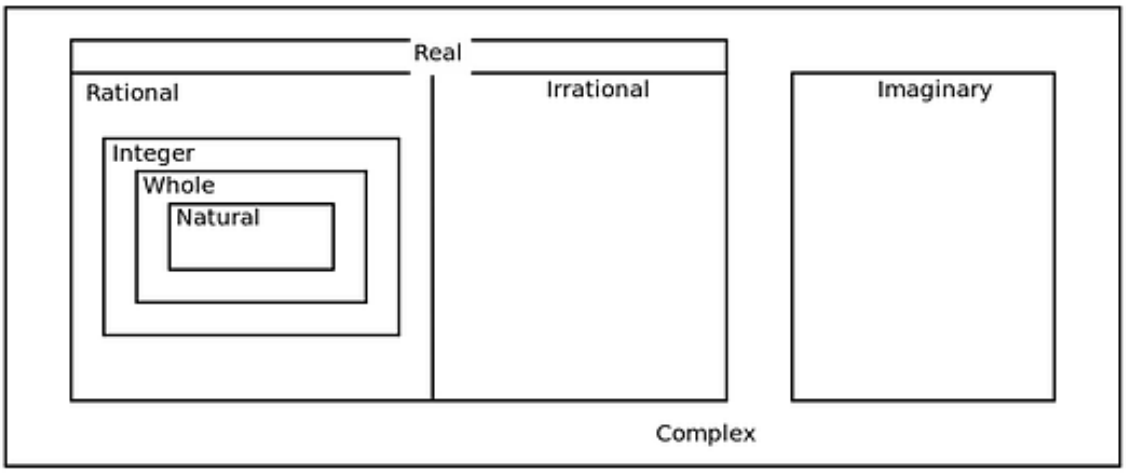
In this lesson, we connected irrational numbers to the measure of geometric figures and showed where a given irrational number is located on the number line. We found irrational solutions of quadratic equations and used the solutions to write the equation in factored form. We also learned of a new set of numbers defined in terms of $\sqrt{-1} = i$ and $i^2 = -1$. We examined how these numbers fit into the number system and performed arithmetic operations on them.

Unit 3, Lesson 6

iNumbers

Jump Start

Complete:



Learning Focus

Support or challenge claims about different types of numbers and the result of adding, subtracting, multiplying, and dividing.

What are the similarities and differences between the arithmetic of integers, rational numbers, real numbers, and complex numbers?

Open Up the Math: Launch, Explore, Discuss

Now that we have been thinking about new number sets, including irrational and complex numbers, it's time to generalize the results.

In order to find solutions to all quadratic equations, we had to extend the number system to include complex numbers.

Do the following for each of the problems below:

- Choose the best word to complete each conjecture.
- After you have made a conjecture, create at least three examples to show why your conjecture is true.
- If you find a counterexample, change your conjecture to fit your work.

1. Conjecture #1: The sum of two integers is [always, sometimes, never] an integer.

More conjectures:

2. The sum of two irrational numbers is [always, sometimes, never] an irrational number.

3. The sum of a rational number and an irrational number is [always, sometimes, never] an irrational number.
4. The difference of two whole numbers is [always, sometimes, never] a whole number.
5. The difference of two rational numbers is [always, sometimes, never] a rational number.
6. The product of two integers is [always, sometimes, never] an integer.
7. The quotient of two integers is [always, sometimes, never] an integer.
8. The product of two rational numbers is [always, sometimes, never] a rational number.
9. The product of two irrational numbers is [always, sometimes, never] an irrational number.
10. The product of two real numbers is [always, sometimes, never] a real number.
11. The product of two complex numbers is [always, sometimes, never] a complex number.
12. The ratio of the circumference of a circle to its diameter is given by the irrational number π. Can the diameter of a circle and the circumference of the same circle both be rational numbers? Explain why or why not.

Ready for More?

Write two claims of your own about number sets and use examples to demonstrate that they are true.

1. Claim #1:
2. Claim #2:

Takeaways

Some true statements about operations on numbers:

Vocabulary

- **closure**

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we examined claims about sets of numbers and the operations of addition, subtraction, multiplication, and division. An example of such a claim is: The quotient of two whole numbers is always a whole number. A counterexample that shows this claim to be false is: $2 \div 7 = \frac{2}{7}$. Since $\frac{2}{7}$ is a rational number, this example shows that dividing two whole numbers does not always result in a whole number.

Unit 3, Lesson 7
Quadratic Quandaries

Learning Focus

Solve quadratic inequalities both graphically and algebraically.

Interpret solutions to quadratic inequalities that arise from context.

How can we use our understanding of solving quadratic equations and graphing quadratic functions to solve quadratic inequalities?

What do our solutions mean in the context of the problem?

How do we identify and write all the solutions?

Open Up the Math: Launch, Explore, Discuss

Carlos and Clarita have a brilliant idea for how they will earn money this summer. Since the community in which they live includes many high schools, a couple of universities, and even some professional sports teams, it seems that everyone has a favorite team they like to root for. In Carlos's and Clarita's neighborhood, these rivalries take on special meaning, since many of the neighbors support different teams. They have observed that their neighbors often display handmade posters and other items to make their support of their favorite team known. The twins believe they can get people in the neighborhood to buy into their new project: painting team logos on curbs or driveways.

For a small fee, Carlos and Clarita will paint the logo of a team on a neighbor's curb, next to their house number. For a larger fee, the twins will paint a mascot on the driveway. Carlos and Clarita have designed stencils to make the painting easier and they have priced the cost of supplies. They have also surveyed neighbors to get a sense of how many people in the community might be interested in purchasing their service.

- Surveys show the twins can sell 100 driveway mascots at a cost of $\$20$, and they will sell 10 fewer mascots for each additional $\$5$ they charge.
- The twins estimate that the cost of supplies will be $\$250$ and they would like to make $\$2,000$ in profit from selling driveway mascots. Therefore, they will need to collect $\$2,250$ in revenue.

This information led Carlos and Clarita to write and solve the quadratic equation:

$(100 - 10x)(20 + 5x) = 2,250$

1. Solve this quadratic equation for x.
2. What do your solutions for x mean in terms of the story context?
3. How would your solution change if this had been the question Carlos and Clarita had asked: "How much should we charge if we want to collect at least $\$2,250$ in revenue?"
4. What about this question: "How much should we charge if we want to maximize our revenue?"

As you probably observed, the situation represented in problem 3 didn't have just one solution, since there are many different prices the twins can charge to collect more than $\$2,250$ in revenue. Sometimes our questions lead to quadratic inequalities rather than quadratic equations.

Here is another quadratic inequality generated by Carlos and Clarita's business plans:

5. Carlos and Clarita want to design a logo that requires less than 48 in^2 of paint and that fits inside a rectangle that is 8 inches longer than it is wide. What are the possible dimensions of the rectangular logo?

Again, problem 5 has multiple answers, and those answers are restricted by the context. Let's examine the inequality you wrote for problem 5, but not restricted by the context.

6. What are the solutions to the inequality $x(x+8) < 48$?

7. How might you support your answer to problem 6 with a graph or a table?

Here are some more quadratic inequalities without contexts. Show how you might use a graph, along with algebra, to solve each of them.

8. $x^2 + 3x - 10 \geq 0$

9. $2x^2 - 5x < 12$

10. $x^2 - 4 \leq 4x + 1$

Carlos and Clarita both used algebra and a graph to solve problem 10, but they both did so in different ways. Illustrate each of their methods with a graph and with algebra.

11. Carlos: "I rewrote the inequality to get 0 on one side and a factored form on the other. I found the zeroes for each of my factors. To decide what values of x made sense in the inequality, I also sketched a graph of the quadratic function that was related to the quadratic expression in my inequality. I shaded solutions for x based on the information from my graph."

12. Clarita: "I graphed a linear function and a quadratic function related to the linear and quadratic expressions in the inequality. From the graph, I could estimate the points of intersection, but to be more exact, I solved the quadratic equation $x^2 - 4 = 4x + 1$ by writing an equivalent equation that had 0 on one side. Once I knew the x-values for the points of intersection in the graph, I could shade solutions for x that made the inequality true."

Ready for More?

Devise a strategy based on your work with quadratic inequalities that could be used to solve this cubic inequality with three factors:

$(x-4)(x+4)(x+8) \leq 0$

Takeaways

Solving quadratic inequalities:

Vocabulary

- **quadratic inequality**

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we developed a strategy for solving quadratic inequalities. The procedure involves solving the related quadratic equation and then using the graph or testing values to find the intervals that are solutions to the inequality. If the inequality represents a real context, the solutions must be interpreted so that they fit the situation.