Unit 2, Lesson 1
Transformers: Shifty y's

Learning Focus

Find patterns in the equations and graphs of quadratic functions.

What happens to the graph of $f(x) = x^2$ when the equation is changed by adding, subtracting, or multiplying by a constant?

Open Up the Math: Launch, Explore, Discuss

Optima Prime is designing a robot quilt for her new grandson. She plans for the robot to have a square face. The amount of fabric that she needs for the face will depend on the area of the face, so Optima decides to model the area of the robot's face mathematically. She knows that the area A of a square with side length x units (which can be inches or centimeters) is modeled by the function $A(x) = x^2$ square units.

1. What is the domain of the function $A(x)$ in this context?

2. Match each statement about the area to the function that models it.

A. The length of each side is increased by 5 units.	1. $A = 5x^2$
B. The length of each side is multiplied by 5 units.	2. $A = (x + 5)^2$
C. The area of a square is increased by 5 square units.	3. $A = (5x)^2$
D. The area of a square is multiplied by 5.	4. $A = x^2 + 5$

3. Optima started thinking about the graph of $y = x^2$ (in the domain of all real numbers). Make a table that contains both negative and positive values of x, and graph the function. What features do you notice?

Optima began wondering about how changes to the equation of the function, like adding 5 or multiplying by 5, affect the graph. She decided to make predictions about the effects and then check them out.

4. Make your own predictions of how the graphs of each of the following equations will be the same or different from the graph of $y = x^2$.

	Similarities to the graph of $y = x^2$	Differences from the graph of $y = x^2$
$y = 5x^2$		
$y = (x + 5)^2$		
$y = (5x)^2$		
$y = x^2 + 5$		

5. Optima decided to test her ideas using technology. She thinks that it is always a good idea to start simple, so she decides to go with $y = x^2 + 5$. She graphs it along with $y = x^2$ in the same window. Test it yourself, and describe what you find.

6. Knowing that things make a lot more sense with more representations, Optima tries a few more examples, like $y = x^2 + 2$ and $y = x^2 - 3$, looking at both a table and a graph for each. What conclusion would you draw about the effect of adding or subtracting a number to $y = x^2$? Carefully record the tables and graphs of these examples, and explain why your conclusion would be true for any value of k, given $y = x^2 + k$.

7. After her amazing success with addition in the last problem, Optima decided to look at what happens with addition and subtraction inside the parentheses, or as she says it, "Adding to the x before it gets squared." Using

your technology, decide the effect of h in the equations $y = (x + h)^2$ and $y = (x - h)^2$. (Choose some specific numbers for h.) Record a few examples (in both tables and graphs) and explain why this effect on the graph occurs.

8. Optima thought that problem 7 was very tricky and had hoped that multiplication was going to be more straightforward. She decides to start simple and multiply by -1, so she begins with $y = -x^2$. Predict what the effect is on the graph, and then test it. Why does it have this effect?

9. Optima is encouraged because she was able to figure out the last problem. She decides to end her investigation for the day by determining the effect of a multiplier, a, in the equation $y = ax^2$. Using both positive and negative numbers, fractions, and integers, create at least two tables and matching graphs to determine the effect of a multiplier.

Ready for More?

Use technology to explore the behavior of the line $y = x$. If the point $(0,0)$ is used as an anchor point, much like $(0,0)$ is used on the graph of the parabola, will the transformations discovered for the parabola work for a line? Why or why not?

Takeaways

Equation	Transformation of $y = x^2$
$y = x^2 + k$	
$y = (x - h)^2$	
$y = (x + h)^2$	
$y = -x^2$	
$y = ax^2$	

Adding Notation, Vocabulary, and Conventions

Vertex: _____

Line of Symmetry: _____

Vocabulary

• angle	• **transformations on a function (non-rigid)**
• **horizontal shift**	• **transformations on a function (rigid)**
• **line of symmetry**	• **vertex**
• parabola	• **vertical shift**
• **reflection**	• **vertical stretch**
• symmetry	

Lesson Summary

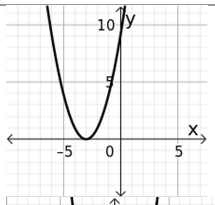
In this lesson, we explored transformations of the function $y = x^2$. We found vertical and horizontal shifts, reflections, and vertical stretches of the parabola. We justified why changes to the equation transform the graph, using tables and our understanding of functions.

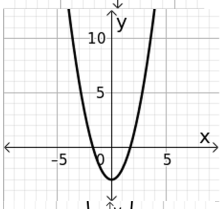
Unit 2, Lesson 2
Transformers: More Than Meets the y's

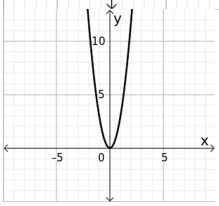
Jump Start

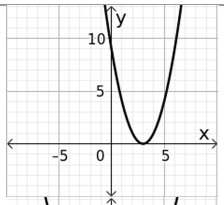
Without using technology, match each of the equations with the graph of the function. Be careful; there are more graphs than equations!

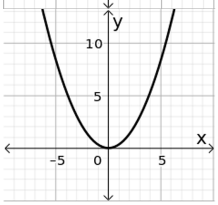
A. $f(x) = x^2 - 3$
B. $f(x) = 3x^2$
C. $f(x) = (x - 3)^2$
D. $f(x) = \frac{1}{3}x^2$

1.


2.


3.


4.


5.


Learning Focus

Write equations for functions that are transformations of $f(x) = x^2$.

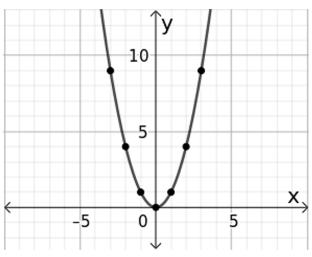
Find efficient methods for graphing transformations of $f(x) = x^2$.

What happens to the graph of $f(x) = x^2$ when more than one transformation is applied?

Open Up the Math: Launch, Explore, Discuss

In the previous lesson, you learned about transformations of the graph of $f(x) = x^2$. In this lesson, you will be writing equations and graphing functions that have more than one transformation. It will be a lot easier if you know a few points on $f(x) = x^2$ that we can use for comparisons. We'll call them anchor points.

1. Identify each anchor point shown on the graph of $f(x) = x^2$.



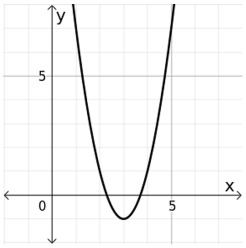
Anchor points:

- Vertex $(_\ ,\ _)$ $(1,\ _)$ and $(-1,\ _)$ $(2,\ _)$ and $(-2,\ _)$ $(3,\ _)$ and $(-3,\ _)$

Line of symmetry: $x =$ _

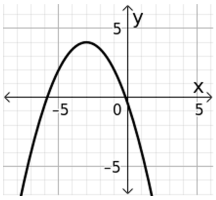
Write the equation for each problem below. Use a second representation to check your equation.

2. The area of a square with side length x, where the side length is decreased by 3, the area is multiplied by 2, and then 4 square units are added to the area.

3. 

4.

x	$f(x)$
-4	7
-3	2
-2	-1
-1	-2
0	-1
1	2
2	7
3	14
4	23

5. 

Graph each equation without using technology. Be sure to have the exact vertex and at least two correct points on either side of the line of symmetry.

6. $f(x) = -x^2 + 3$

7. $g(x) = (x + 2)^2 - 5$

8. $h(x) = 3(x - 1)^2 + 2$

9. Given: $f(x) = a(x - h)^2 + k$

    a. What point is the vertex of the parabola?

    b. What is the equation of the line of symmetry?

    c. How can you tell if the parabola opens up or down?

    d. How do you identify the vertical stretch?

10. Does it matter in which order the transformations are done? Explain why or why not.

Ready for More?

Think about applying the transformations to the parent function $f(x) = 2^x$.

1. What point makes sense to use as an anchor point on this function?
2. What do you think is the equation of the function with a horizontal shift left 3?
3. How does the horizontal shift on $y = 2^x$ work like the horizontal shift on $y = x^2$?

Takeaways

Vertex form of a quadratic equation: _____

• Vertex: _____ • Line of symmetry: _____ • Vertical stretch: _____	• Opens upward: _____ • Opens downward: _____

Quick-graph method for graphing quadratics:

a. $y = (x - 4)^2 - 3$

b. $y = -(x + 2)^2 + 5$

c. $y = 2(x + 1)^2 - 5$

Vocabulary

- **vertex form**

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we learned to graph quadratic functions that have a combination of transformations. We found that the vertex form of the equation of a quadratic function makes it easy to find the vertex and identify the transformations. We wrote equations in vertex form from graphs and tables, using our understanding of transformations and the features of parabolas.

Unit 2, Lesson 3
Building the Perfect Square

Jump Start

Graph each function.

1. $y = -x^2 + 5$
2. $y = (x - 1)^2 - 3$
3. $y = 2x^2 - 5$

Learning Focus

Find the square of a binomial expression.

Recognize a perfect square trinomial.

Create perfect squares from partial areas.

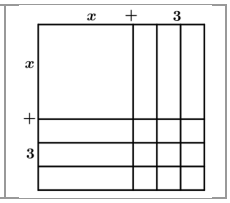
Find relationships between terms in a perfect square trinomial.

How can we use models to find equivalent expressions for perfect squares?

Open Up the Math: Launch, Explore, Discuss

Optima has a quilt shop where she sells many colorful quilt blocks for people who want to make their own quilts. She has quilt designs that are made so that they can be sized to fit any bed. She bases her designs on quilt squares that can vary in size, so she calls the length of the side for the basic square x, and the area of the basic square is the function $A(x) = x^2$. In this way, she can customize the designs by making bigger squares or smaller squares.

1. If Optima adds 3 inches to the side of the square, what is the area of the square?

When Optima draws a pattern for the square in problem 1, it looks like this:	

2. Use both the diagram and the equation $A(x) = (x + 3)^2$ to explain why the area of the quilt block square, $A(x)$, is also equal to $x^2 + 6x + 9$.

The customer service representatives at Optima's shop work with customers and write up the orders based on the area of the fabric needed. As you can see from problem 2, there are two ways that customers can call in and describe the area of the quilt block. One way describes the length of the sides of the block, and the other way describes the areas of each of the 4 sections of the block.

For each of the following quilt blocks, draw the diagram of the block and write two equivalent equations for the area of the block.

3. Block with side length $x + 2$.
4. Block with side length $x + 1$.
5. What patterns do you notice when you relate the diagrams to the two expressions for the area?
6. Optima likes to have her little dog, Clementine, around the shop. One day, Clementine got a little hungry, and started to chew up the orders. When Optima found the orders, one of them was so chewed up that there were only partial

expressions for the area remaining. Help Optima by completing each of the following expressions for the area so that they describe a perfect square. Then, write the two equivalent equations for the area of the square.

a. $x^2 + 4x$

b. $x^2 + 6x$

c. $x^2 + 8x$

d. $x^2 + 12x$

7.

a. If $x^2 + bx + c$ is a perfect square, what is the relationship between b and c?

b. What process can be used to find c and complete the square, given $x^2 + bx$?

c. Will this strategy work if b is negative? Why or why not?

d. Will the strategy work if b is an odd number? What happens to c if b is odd?

8. One of the new customer service representatives thinks she doesn't need to draw diagrams anymore because she found a great shortcut. She writes $(x + 5)^2 = x^2 + 25$. Examine her thinking. Is she right, or does she need to revise her strategy? What suggestions would you make, and how would they help?

Ready for More?

Prove that $(x + h)^2 = x^2 + 2xh + h^2$, either algebraically or using a diagram.

Takeaways

If $x^2 + bx + c$ is a perfect square, _____

The square of a binomial: _____

An example is: _____

Adding Notation, Vocabulary, and Conventions

Binomial: _____ Trinomial: _____ Squaring a binomial: _____	

Vocabulary

- **binomial**
- **completing the square**
- **trinomial**

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we connected area models for multiplication to show how to multiply binomials to get a perfect square trinomial. We learned to recognize a perfect square trinomial by looking for a relationship between the second and third terms. We also worked to create a perfect square when given the first two terms of a trinomial.

Unit 2, Lesson 4

Square Deal

Learning Focus

Find a process for completing the square that works on all quadratic functions.

Adapt diagrams to become more efficient in completing the square.

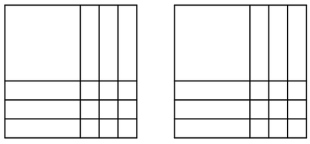
How can we complete the square when there is more than one square given (or $a \neq 1$ in $ax^2 + bx + c$)?

Open Up the Math: Launch, Explore, Discuss

Remember Optima's quilt shop? She bases her designs on quilt squares that can vary in size, so she calls the length of the side for the basic square x, and the area of the basic square is the function $A(x) = x^2$. In this way, she can customize the designs by making bigger squares or smaller squares.

1. Sometimes a customer orders more than one quilt block of a given size. For instance, when a customer orders 4 blocks of the basic size, the customer service representatives write up an order for $A(x) = 4x^2$. Model this area expression with a diagram.

2. One of the customer service representatives finds an envelope that contains the blocks pictured below. Write the order that shows two equivalent equations for the area of the blocks.



3. What equations for the area could customer service write if they received an order for 2 square blocks that have both dimensions increased by 1 inch in comparison to the basic block? Write the area equations in two equivalent forms. Verify your algebra using a diagram.

4. What if customer service receives an order for 3 square blocks with both dimensions increased by 2 inches in comparison to the basic block? Again, show two different equations for the area and verify your work with a model.

5. Clementine is at it again! When is that dog going to learn not to chew up the orders? (She also chews Optima's shoes, but that's a story for another day.) Here are some of the orders that have been chewed up so they are missing the last term. Help Optima by completing each of the following expressions for the area so that they describe a perfect square. Then, write the two equivalent equations for the area of the square.

    a. $2x^2 + 8x$

    b. $3x^2 + 24x$

Sometimes the quilt shop gets an order that turns out to be more or less than a perfect square. Customer service always tries to fill orders with perfect squares, or at least, they start there and then adjust as needed. They always write their equations in a way that relates the area to the closest perfect square.

6. Now here's a real mess! Customer service received an order for an area $A(x) = 2x^2 + 12x + 13$. Help them to figure out an equivalent expression for the area using the diagram.



7. Optima really needs to get organized. Here's another scrambled diagram. Write two equivalent equations for the area of this diagram.



8. Optima realized that not everyone needs perfect squares and not all orders are coming in as expressions that are perfect squares. Determine whether each expression below is a perfect square and why the expression is or is not a perfect square. If it is not a perfect square, find the perfect square that seems "closest" to the given expression, and show how the perfect square can be adjusted to be equivalent to the given expression.

   a. $A(x) = x^2 + 6x + 13$

   b. $A(x) = x^2 - 8x + 16$

   c. $A(x) = x^2 - 10x - 3$

   d. $A(x) = 2x^2 + 8x + 14$

   e. $A(x) = 3x^2 - 30x + 75$

   f. $A(x) = 2x^2 - 22x + 11$

9. Now let's generalize. Given an expression in the form $ax^2 + bx + c \ (a \neq 0)$, describe a step-by-step process for completing the square.

Ready for More?

Use the completing the square process on the following expression:

$\frac{1}{2}x^2 + 4x + 8$

Takeaways

Completing the square for $y = ax^2 + bx + c$ when $a = 1$

Completing the square for $y = ax^2 + bx + c$ when $a \neq 1$

Lesson Summary

In this lesson, we solidified a process for completing the square with expressions in the form $ax^2 + bx + c$ with $a \neq 1$. We learned an algebraic procedure that goes along with an open diagram that supports our work. We also verified that the expression obtained by completing the square was equivalent to the original expression using the distributive property.

Unit 2, Lesson 5

Be There or Be Square

Jump Start

Graph each function with at least two precise points on either side of the line of symmetry.

1. $y = 2(x + 3)^2 - 5$
2. $y = -2(x - 3)^2 + 5$

Learning Focus

Use completing the square to change the form of a quadratic equation.

Graph quadratic equations given in standard form.

How might completing the square help us to graph parabolas?

Open Up the Math: Launch, Explore, Discuss

Quilts and Quadratic Graphs

Optima's niece, Jenny, works in the shop, taking orders and drawing quilt diagrams. When the shop isn't too busy, Jenny pulls out her math homework and works on it. One day, she is working on graphing parabolas and notices that the equations she is working with look a lot like an order for a quilt block. For instance, Jenny is supposed to graph the equation: $y = (x - 3)^2 + 4$. She thinks, "That's funny. This would be an order where the length of the standard square is reduced by 3 and then we add a little piece of fabric that has as area of 4. We don't usually get orders like that, but it still makes sense. I better get back to thinking about parabolas. Hmmm…."

1. Fully describe the parabola that Jenny has been assigned to graph.

2. Jenny returns to her homework, which is about graphing quadratic functions. Much to her dismay, she finds that she has been given: $y = x^2 - 6x + 9$. "Oh dear," thinks Jenny. "I can't tell where the vertex is, or identify any of the transformations of the parabola in this form. Now what am I supposed to do?"

   "Wait a minute—is this the area of a perfect square?" Use your work from Lesson 3, Building the Perfect Square, to answer Jenny's question and justify your answer.

3. Jenny says, "I think I've figured out how to change the form of my quadratic equation so that I can graph the parabola. I'll check to see if I can make my equation a perfect square." Jenny's equation is: $y = x^2 - 6x + 9$.

   See if you can change the form of the equation, find the vertex, and graph the parabola.

   a. $y = x^2 - 6x + 9$

      New form of the equation:

   b. Vertex of the parabola:

   c. Graph (with at least 3 precise points on each side of the line of symmetry):

4. The next quadratic to graph from Jenny's homework is $y = x^2 + 4x + 2$. Does this expression fit the pattern for a perfect square? Why or why not?

    a. Complete the square so that the equation can be written in vertex form, $y = a(x - h)^2 + k$.

    b. Is the equation you have written equivalent to the original equation? If not, what adjustments need to be made? Why?

    c. Identify the vertex, and graph the parabola with three precise points on both sides of the line of symmetry.

5. Jenny had hoped that she wasn't going to need to figure out how to complete the square on an equation where b is an odd number. Of course, that was the next problem. Help Jenny to find the vertex of the parabola for this quadratic function:

    $g(x) = x^2 + 7x + 10$

6. Don't worry if you had to think hard about problem 5. Jenny needs to do a couple more:

    a. $g(x) = x^2 - 5x + 3$

    b. $g(x) = x^2 - x - 5$

7. It just gets better! Help Jenny find the vertex, and graph the parabola for the quadratic function $h(x) = 2x^2 - 12x + 17$.

8. This one is just too cute—you've got to try it! Find the vertex, and describe the parabola that is the graph of $f(x) = \frac{1}{2}x^2 + 2x - 3$.

Ready for More?

Use technology to experiment with graphing a quadratic function in standard form: $y = ax^2 + bx + c$. Check out the effect of changing a, b, or c. Are there patterns you can find that help you predict if the parabola opens up or down? Can you predict intercepts or the location of the vertex.

Takeaways

A method for graphing a quadratic equation in standard form: $y = ax^2 + bx + c$.

Vocabulary

- **standard form of a quadratic function**
- vertex form

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we learned to graph a quadratic function in standard form. We used the process of completing the square to help identify the transformations and locate the vertex. From there, we were able to use the quick-graph method to graph the parabola.

Unit 2, Lesson 9

Lining Up Quadratics

Learning Focus

Find patterns to efficiently graph quadratic functions from factored form.

What features of a parabola are highlighted in factored form? How can we use those features to graph a quadratic function?

How does the factored form of a quadratic equation relate to graphing a parabola?

Open Up the Math: Launch, Explore, Discuss

Use technology to graph each function and find the vertex, the line of symmetry, the y-intercept, and the x-intercepts. Be sure to properly write the intercepts as points.

1. $y = (x - 1)(x + 3)$
2. $f(x) = 2(x - 2)(x - 6)$
3. $g(x) = -x(x + 4)$
4. Based on these examples, how can you use a quadratic function in factored form to:
    a. Find the line of symmetry of the parabola?
    b. Find the vertex of the parabola?
    c. Find the x-intercepts of the parabola?
    d. Find the y-intercept of the parabola?
    e. Find the vertical stretch?

Now it's time to try your strategy! Factor each of the functions, use the strategy that you found in problem 4 to find the vertex, intercepts, vertical stretch, and line of symmetry, and graph each parabola without technology. Check your work with technology, and if your graph is wrong, go back and examine each step of your work to diagnose the problem.

5. $f(x) = x^2 - 2x - 8$
6. $f(x) = -x^2 - 6x - 5$
7. $f(x) = 2x^2 - 12x + 16$
8. $f(x) = -2x^2 - 5x + 3$

Ready for More?

Write three functions in factored form with a line of symmetry $x = 2$. For each function, find the vertex and the intercepts.

Takeaways

When a quadratic function is in factored form $f(x) = a(x + m)(x + n)$:

The x-intercepts can be found by _____

The y-intercept can be found by _____

The line of symmetry can be found by _____

The vertex can be found by _____

The vertical stretch and reflection can be found by _____

Vocabulary

- **factored form**

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we learned to use the factored form of a quadratic equation to graph parabolas. We learned to find the x-intercepts from the factors, then find the line of symmetry between the x-intercepts. Once we knew the line of symmetry, we could find the vertex. We observed several patterns that helped to make factored form an efficient way to graph quadratics.

Unit 2, Lesson 10

I've Got a Fill-in

Learning Focus

Choose the most efficient form of a quadratic function.

Become efficient and accurate in converting from one quadratic form to another.

Become efficient and accurate in identifying features of the graph of quadratic functions from a given form.

What information do we get from each form of a quadratic equation, and which form is best for a particular purpose?

Open Up the Math: Launch, Explore, Discuss

For each problem below, you are given a piece of information that tells you a lot. Use what you know about that information to fill in the rest.

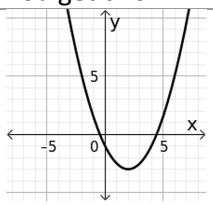
1. Complete:

You get this:	Fill in this:
$y = x^2 - x - 12$	Factored form of the equation:
	Graph of the equation:

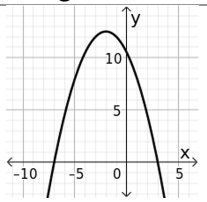
2. Complete:

You get this:	Fill in this:
$y = x^2 - 6x + 3$	Vertex form of the equation:
	Graph of the equation:

3. Complete:

You get this:	Fill in this:
	Vertex form of the equation:
	Standard form of the equation:

4. Complete:

You get this:	Fill in this:
	Factored form of the equation:
	Standard form of the equation:

5. Complete:

You get this:	Fill in this:
$y = -x^2 - 6x + 16$	Either form of the equation other than standard form:
	Vertex of the parabola:
	x-intercepts and y-intercept:

6. Complete:

You get this:	Fill in this:
$y = 2x^2 + 12x + 13$	Either form of the equation other than standard form:
	Vertex of the parabola:
	x-intercepts and y-intercept:

7. Complete:

You get this:	Fill in this:
$y = -2x^2 + 14x + 60$	Either form of the equation other than standard form:
	Vertex of the parabola:
	x-intercepts and y-intercept:

Ready for More?

Play the I've Got A Fill-in game. You need four people who are finished with the task to play. The first person starts by writing a function in factored form. Don't make the numbers too messy. The game continues with the next person filling in block 2, the third person completing block 3, and so on. Each time the next person starts completing their block, the other people in the group should be checking the previous work. If you can correct an error, you get extra brownie points. Have fun!

1: Write a quadratic function in factored form.	2: Find x-intercepts, the y-intercept, the vertex, and the line of symmetry.
3: Write the function in standard form.	4: Graph the function.

Takeaways

It is easier to convert to vertex form when: _____

Vertex form is efficient for finding: _____

It is easier to convert to factored form when: _____

Factored form is efficient for finding: _____

Standard form is efficient for finding: _____

Helpful hints for avoiding algebraic errors in factoring, completing the square, or changing to standard form:

Lesson Summary

In this lesson, we learned to make strategic choices about the most efficient form for working with the graph of a quadratic function. We considered which form is most efficient for obtaining features like the vertex, x-intercepts, y-intercept, the vertical stretch, and reflection. We also considered which form will be most efficient to convert from standard form, knowing that some trinomials do not factor easily and some trinomials make completing the square complicated.