Unit 8, Lesson 1

Go the Distance

Learning Focus

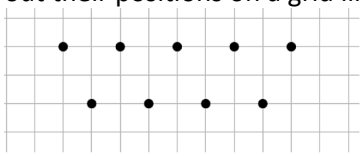
Find the distance between two points in the coordinate plane.

Find the perimeter of a geometric figure in the coordinate plane.

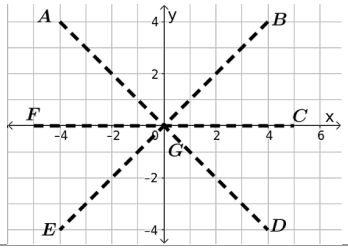
How can I find the distance between two points if they're not on vertical or horizontal lines?

Open Up the Math: Launch, Explore, Discuss

The performances of the Crawford High School drill team are very popular during half-time at the school's football and basketball games. When the Crawford High School drill team choreographs the dance moves that they will do on the football field, they lay out their positions on a grid like the one shown:



In one of their dances, they plan to make patterns holding long, wide ribbons that will span from one dancer in the middle to six other dancers. On the grid, their pattern looks like this:

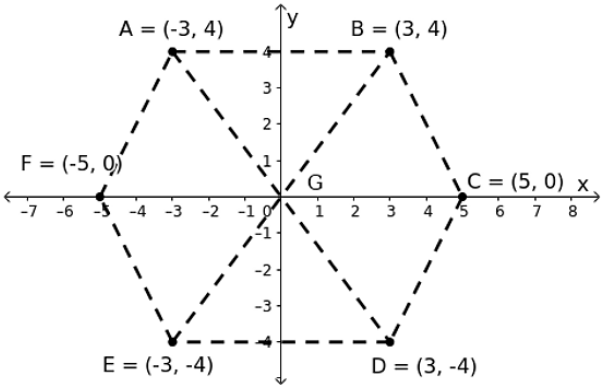


The question the dancers have is how long to make the ribbons. Gabriela (G) is standing in the center, and some dancers think that the ribbon from Gabriela (G) to Courtney (C) will be shorter than the one from Gabriela (G) to Brittney (B).

1. How long does each ribbon need to be?

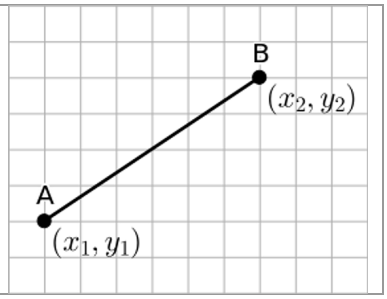
2. Explain how you found the length of each ribbon.

When they have finished with the ribbons in this position, they are considering using them to form a new pattern like this:



3. Will the ribbons they used in the previous pattern be long enough to go between Brittney (B) and Courtney (C) in the new pattern? Explain your answer.

Gabriela notices that the calculations she is making for the length of the ribbons remind her of math class. She says to the group, "Hey, I wonder if there is a process that we could use like what we have been doing to find the distance between any two points on the grid." She decides to think about it like this:



"I'm going to start with two points and draw the line between them that represents the distance that I'm looking for. Since these two points could be anywhere, I named them $A(x_1, y_1)$ and $B(x_2, y_2)$. Hmmmmm. . . when I figured the length of the ribbons, what did I do next?"

4. Think back on the process you used to find the length of the ribbon and write down your steps here, in terms of (x_1, y_1) and (x_2, y_2).

5. Use the process you came up with in problem 4 to find the distance between two points located far enough away from each other that using your formula from problem 4 is more efficient than graphing and counting. For example, find the distance between $(-11,25)$ and $(23, -16)$.

6. Use your process to find the perimeter of the hexagon pattern shown in problem 3.

Ready for More?

Find as many points as you can that are 3 units away from the point $(1,1)$. Plot each of the points along with $(1,1)$. What do you notice about the graph of the points?

Takeaways

Steps in words	Steps in symbols

The Distance Formula:

Finding the perimeter of a geometric figure on the coordinate plane: _____

Lesson Summary

In this lesson, we learned to find the distance between two points. We used the Pythagorean theorem to develop a formula that could be used whenever we need to find the length of a segment between two points. The formula can be applied to find the length of the sides of a geometric figure in the coordinate plane to calculate the perimeter.

Unit 8, Lesson 2
Yard Work in Segments

Learning Focus

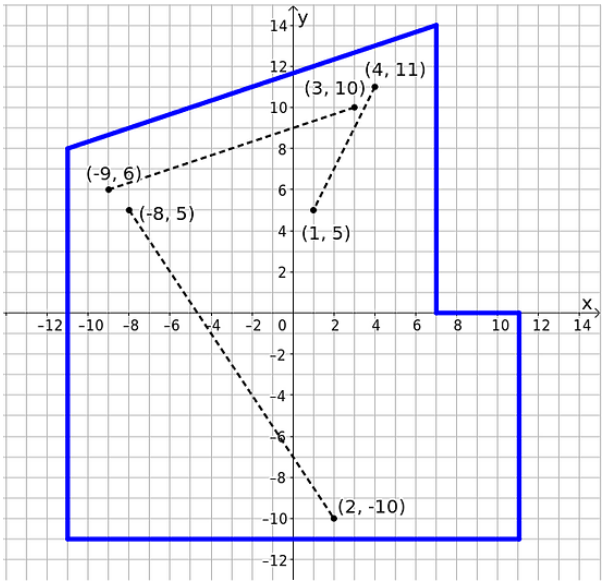
Locate the midpoint of a segment and a point that divides the segment in a given ratio.

How do I locate the midpoint of a segment given just the coordinates of its endpoints?

How do I divide a line segment drawn on a grid into proportional parts?

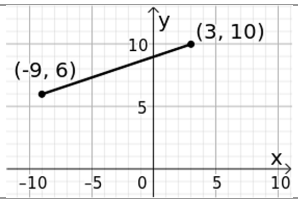
Open Up the Math: Launch, Explore, Discuss

Malik's family has purchased a new house with an unfinished yard. They drew the following map of the backyard:



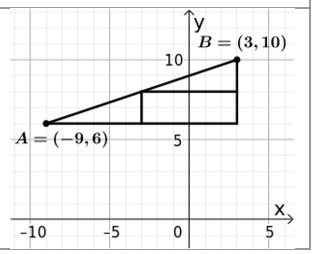
Malik and his family are using the map to set up gardens and patios for the yard. They plan to lay out the yard with stakes and strings (shown as points and dotted lines in the map) so they know where to plant grass, flowers, or vegetables. They want to begin with a vegetable garden that will be parallel to the fence shown at the top of the map.

1. They set the first stake at $(-9,6)$ and the stake at the end of the garden at $(3,10)$. They want to mark the middle of the garden with another stake. Where should the stake that is at the midpoint of the segment between the two end stakes be located? Using a diagram, describe your strategy for finding this point.



Midpoint:

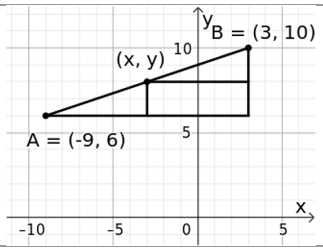
Malik figured out the midpoint by saying, "It makes sense to me that the midpoint is going to be halfway over and halfway up, so I drew a right triangle and cut the horizontal side in half and the vertical side in half like this:"
Malik continued, "That put me right at $(-3,8)$. The only thing that seems funny about that to me is that I know the base of the big triangle was 12 and the height of the triangle was 4, so I thought the midpoint might be $(6,2)$."



2. Explain to Malik why the logic that made him think the midpoint was $(6,2)$ is almost right, and how to extend his thinking to use the coordinates of the endpoints to get the midpoint of $(-3,8)$.

Malik's sister, Sapana, looked at his drawing and said, "Hey, I drew the same picture, but I noticed the two smaller triangles that were formed were congruent. Since I didn't know for sure what the midpoint was, I called it (x, y). Then I used that point to write an expression for the length of the sides of the small triangles. For instance, I figured that the base of the lower triangle was $x-(-9)$."

3. Label all of the other legs of the two smaller right triangles using Sapana's strategy.



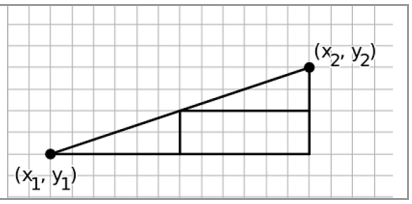
Sapana continued, "Once I labeled the triangles, I wrote equations by making the bases equal and the heights equal."

4. Does Sapana's strategy work? Show why or why not.

5. Choose a strategy and use it to find the midpoint of the segment with endpoints $(-3,4)$ and $(2,9)$.

6.

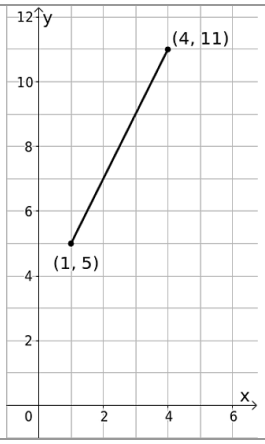
Use either strategy to find the midpoint of the segment between (x_1, y_1) and (x_2, y_2).



Pause and Reflect

The next area in the garden to be marked is for a flower garden. Malik's parents have the idea that part of the garden should contain a big rose bush and the rest of the garden will have smaller flowers like petunias. They want the section with the other flowers to be twice as long as the section with the rose bush. The stake on the endpoints of this garden will be at $(1,5)$ and $(4,11)$. Malik's dad says, "We'll need a stake that marks the end of the rose garden."

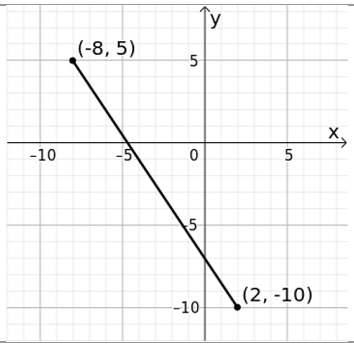
7. Help Malik and Sapana figure out where the stake will be located if the rose bush will be closer to the stake at $(1,5)$ than the stake at $(4,11)$.



The stake should be located at:

There's only one more set of stakes to put in. This time the endpoint stakes are at $(-8,5)$ and $(2,-10)$. Another stake needs to be placed that partitions this segment into two parts so that the ratio of the lengths is $2:3$.

8. Where must the stake be located if it is to be closer to the stake at $(-8,5)$ than to the stake at $(2,-10)$?

	The stake should be located at:

Ready for More?

Generate a symbolic rule for locating the point that divides a line segment into two parts so that the ratio of the lengths is $m:n$, with the point closer to the left endpoint.

Takeaways

To find the midpoint of the segment whose endpoints are (x_1, y_1) and (x_2, y_2), I can use the midpoint rule:

To find the point that divides the segment whose endpoints are (x_1, y_1) and (x_2, y_2) into two parts in the ratio $m:n$, I can use a strategic method or remember the rule:

Strategy:

Rule: _____

Here are my illustration and notes to explain why this rule or strategy works:

Vocabulary

- directed distance
- **midpoint**
- ratio

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we examined strategies for dividing a line segment into two parts that fit a given ratio. One common application of this concept is to find the coordinates of the midpoint of a segment, given the coordinates of the endpoints.

Unit 8, Lesson 3

Slippery Slopes

Learning Focus

Prove slope relationships between parallel lines and perpendicular lines.

How do we know that the slopes of any two perpendicular lines are negative reciprocals? Is it always true or sometimes true?

Open Up the Math: Launch, Explore, Discuss

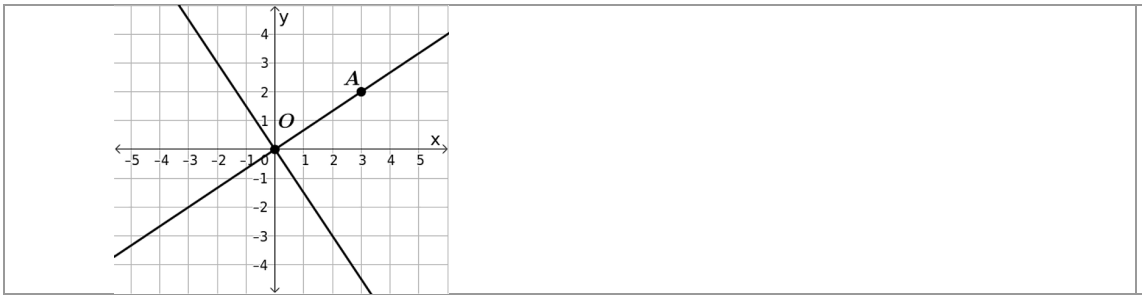
While working with right triangles previously, you examined several examples that led to the conclusion that the slopes of perpendicular lines are negative, or opposite, reciprocals. Your goal now is to formalize this work into a proof. Let's start by thinking about two perpendicular lines that intersect at the origin, like these:

1.

   a. Start by drawing a right triangle with the segment $\overline{OA}$ as the hypotenuse. You'll need a vertical segment, $\overline{AB}$, from point A to point B on the x-axis and a horizontal segment from point B to point O. These are often called slope triangles. Based on the slope triangle that you have drawn, what is the slope of $\overleftrightarrow{OA}$?

   b. Now, rotate the slope triangle 90° counterclockwise about the origin. What are the coordinates of the image of point A?

   c. Using this new point, A', draw a slope triangle with hypotenuse $\overline{OA'}$. Based on the slope triangle, what is the slope of the line $\overleftrightarrow{OA'}$?



2. What is the relationship between these two slopes? How do you know?

3. Is the relationship changed if the two lines are translated so that the intersection is at $(-5,7)$?

   How do you know?

To prove a theorem, we need to demonstrate that the property holds for any pair of perpendicular lines, not just a few specific examples. It is often done by drawing a very similar picture to the examples we have tried, but using variables instead of numbers. Using variables represents the idea that it doesn't matter which numbers we use, the relationship stays the same. Let's try that strategy with the theorem about perpendicular lines having slopes that are negative reciprocals.

4.

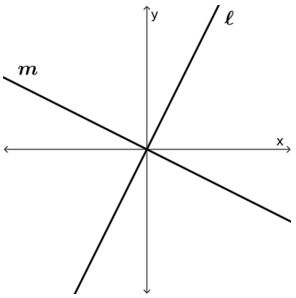
a. Lines ℓ and m are constructed to be perpendicular.

b. Start by labeling a point P on the line ℓ.

c. Label the coordinates of P.

d. Draw the slope triangle from point P.

e. Label the lengths of the sides of the slope triangle using variables like a and b for the run and the rise.



5. What is the slope of line ℓ?

Rotate point P $90°$ about the origin, label it P', and mark it on line m. What are the coordinates of P'?

6. Draw the slope triangle from point P'. What are the lengths of the sides of the slope triangle? How do you know?

7. What is the slope of line m?

8. What is the relationship between the slopes of line ℓ and line m? How do you know?

9. Is the relationship between the slopes changed if the intersection between line ℓ and line m is translated to another location? How do you know?

10. Is the relationship between the slopes changed if lines ℓ and m are rotated?

11. How do these steps demonstrate that the slopes of perpendicular lines are negative reciprocals for any pair of perpendicular lines?

In problems 6–9, you proved that the product of the slopes of two perpendicular lines intersecting at the origin is -1. You have also observed that if the intersection of the two perpendicular lines is not $(0,0)$, you can translate the intersection to the origin. By definition, all the angles and distances are preserved under such a translation. From there, the relationship that the product of the slopes is -1, as described above, would still be the same.

12. Write a proof of the theorem: If two lines are perpendicular, the slopes are negative reciprocals.

    Given: $\ell \perp m$

    Let q_1 be the slope of ℓ and q_2 be the slope of m.

    Prove: $q_1 = -\frac{1}{q_2}$

Now think about parallel lines like the ones shown: $m \parallel n$.

Points O, P, and Q all lie on line n. Point B is the y-intercept of line m.

13.

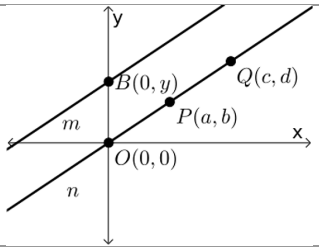
a. Draw the slope triangle from point P to point Q. What is the slope of $\overleftrightarrow{PQ}$?

b. Translate line n so that point O coincides with point B. Why will P' and Q' lie on line m?

c. Draw the slope triangle from point P' to point Q'. What are the coordinates of P' and Q'?

d. What is the slope of $\overleftrightarrow{P'Q'}$?

e. Show how you know that these two parallel lines have the same slope, and explain why this proves that all parallel lines have the same slope.



Ready for More?

Find the equation of the perpendicular bisector of the segment with endpoints $(-5,6)$ and $(4,-3)$.

Takeaways

Given: $m \parallel n$ with point O with coordinates $(0,0)$, point P with coordinates (a, b), and point Q with coordinates (c, d) on n. Let q_1 be the slope of n and q_2 be the slope of m.

Prove: $q_1 = q_2$

Lesson Summary

In this lesson, we used transformations to prove that the slopes of perpendicular lines are negative reciprocals and the slopes of parallel lines are equal. To prove the theorems, we needed to write the lines and points so that they were general enough to cover all cases. When we used a specific point like the origin, we needed to make an argument that the relationship could be extended to any pair of lines that are parallel or perpendicular.

Unit 8, Lesson 4

Prove It with Algebra

Learning Focus

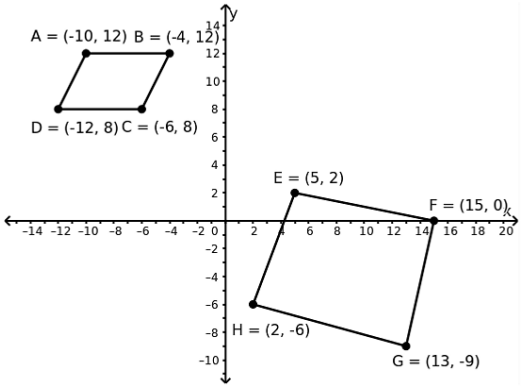
Prove quadrilaterals are parallelograms, rectangles, rhombi, or squares using coordinates.

Find the perimeter and area of a quadrilateral on the coordinate plane.

How do I use algebra to show that a quadrilateral is a parallelogram, a rectangle, a rhombus, or a square?

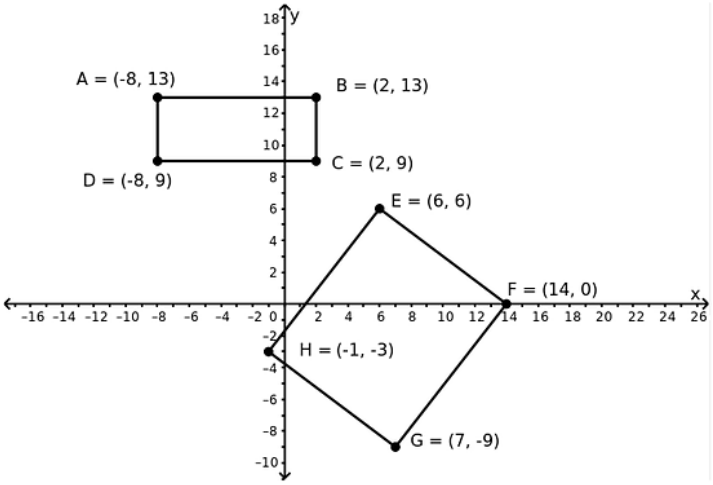
Open Up the Math: Launch, Explore, Discuss

In this task you need to use all the things you know about quadrilaterals, distance, and slope to prove that the shapes are parallelograms, rectangles, rhombi, or squares. Be systematic and be sure that you give all the evidence necessary to verify your claim.

1. 

   a. Is $ABCD$ a parallelogram? Explain how you know.

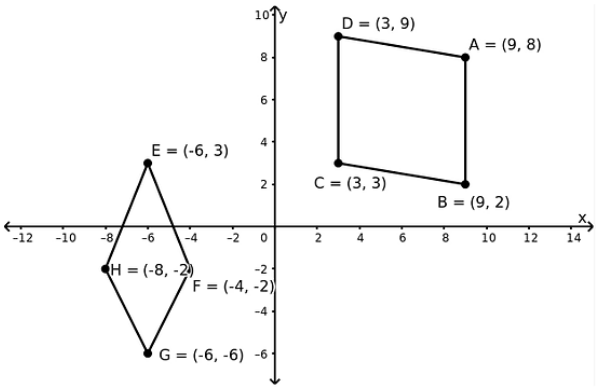
   b. Is $EFGH$ a parallelogram? Explain how you know.

2. 

   a. Is $ABCD$ a rectangle? Explain how you know.

   b. Is $EFGH$ a rectangle? Explain how you know.

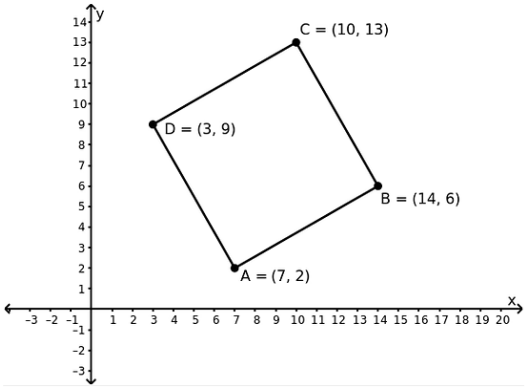
3.



a. Is $ABCD$ a rhombus? Explain how you know.

b. Is $EFGH$ a rhombus? Explain how you know.

4.



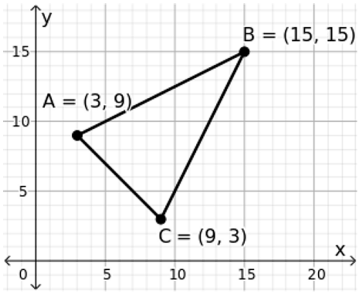
a. Is $ABCD$ a square? Explain how you know.

b. Find the perimeter and area of the quadrilateral.

5.

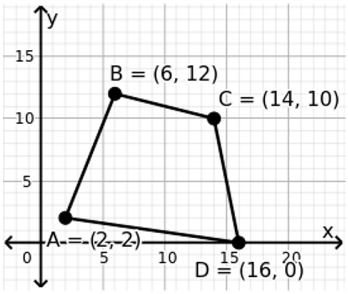
a. Find the midpoint of side AB and side BC of the triangle. Label these midpoints M and N. What relationship exists between segment MN and side AC of the triangle? Explain how you know.

b. Now find the point $\frac{1}{3}$ of the distance from A to B and $\frac{1}{3}$ of the distance from C to B in the triangle. Label these points R and S. What relationship exists between segment RS and side AC of the triangle? Explain how you know.



Ready for More?

Find the midpoints of each of the sides of quadrilateral $ABCD$ and label the midpoints M, N, P, and Q. Figure $MNPQ$ is what type of quadrilateral? How do you know?



Takeaways

Ways to use coordinates to prove quadrilaterals are parallelograms, rectangles, rhombi, or squares:

Lesson Summary

In this lesson, we used the distance formula, the midpoint rule, and the properties of slopes of parallel and perpendicular lines to determine if a given set of 4 points on a coordinate plane formed the vertices of a parallelogram, rectangle, rhombus, or square.