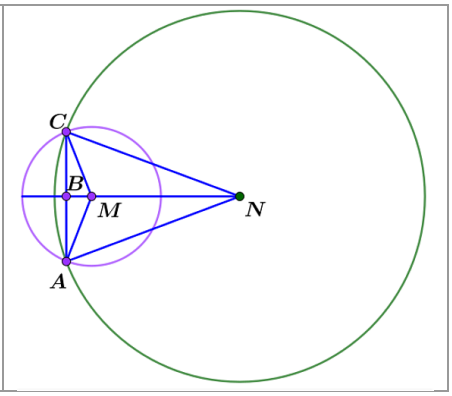
Unit 7, Lesson 1

Under Construction

Jump Start

Examine the diagram. (Note: points M and N are at the center of their respective circles.)	

1. Write as many equality statements as you can to represent equal lengths in the diagram.
2. Write as many congruence statements as you can to represent congruent segments in the diagram.
3. Be prepared to state how you know which segments are congruent. Are there any segments you are wondering about, but aren't sure if they are really congruent?

Learning Focus

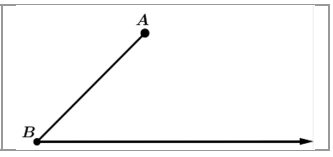
Construct a rhombus, a perpendicular bisector, and a square using only a compass and a straightedge (unmarked ruler) as tools.

How do I use geometric objects, such as circles and lines, to construct geometric figures like rhombuses and squares, rather than using measurement tools, such as rulers and protractors, to draw such figures?

Open Up the Math: Launch, Explore, Discuss

In ancient times, one of the only tools builders and surveyors had for laying out a plot of land or the foundation of a building was a piece of rope.

There are two geometric figures you can create with a piece of rope: you can pull it tight to create a line segment, or you can fix one end, and—while extending the rope to its full length—trace out a circle with the other end. Geometric constructions have traditionally mimicked these two processes using an unmarked straightedge to create a line segment and a compass to trace out a circle (or sometimes a portion of a circle called an arc). Using only these two tools, you can construct all kinds of geometric shapes.

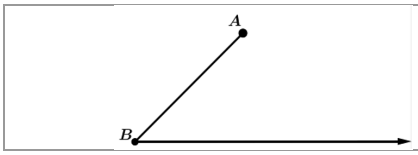
Suppose you want to construct a rhombus using only a compass and straightedge. You might begin by drawing a line segment to define the length of a side, and drawing another ray from one of the endpoints of the line segment to define an angle, as in the sketch.	

Now the hard work begins. We can't just keep drawing line segments because we have to be sure that all four sides of the rhombus are the same length. We have to stop drawing and start constructing.

Construct a Rhombus

Knowing what you know about circles and line segments, how might you locate point C on the ray in the diagram given, so the distance from B to C is the same as the distance from B to A?

1. Describe how you will locate point C and how you know $\overline{BC} \cong \overline{BA}$, then construct point C on the diagram given.



Now that we have three of the four vertices of the rhombus, we need to locate point D, the fourth vertex.

2. Describe how you will locate point D and how you will know $\overline{CD} \cong \overline{DA} \cong \overline{AB}$, then construct point D on the diagram.

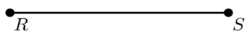
Pause and Reflect

Construct a Perpendicular Bisector of a Segment and a Square (a rhombus with right angles)

The only difference between constructing a rhombus and constructing a square is that a square contains right angles. Therefore, we need a way to construct perpendicular lines using only a compass and a straightedge.

We will begin by inventing a way to construct a perpendicular bisector of a line segment.

3. Given $\overline{RS}$, fold and crease the paper so that point R is reflected onto point S. Based on the definition of reflection, what do you know about this "crease line"?



You have "constructed" a perpendicular bisector of $\overline{RS}$ by using a paper-folding strategy. Is there a way to construct this line using a compass and a straightedge?

4. Experiment with the compass to see if you can develop a strategy to locate points on the "crease line." When you have located at least two points on the "crease line," use the straightedge to finish your construction of the perpendicular bisector. Describe your strategy for locating points on the perpendicular bisector of $\overline{RS}$.

Now that you have created a line perpendicular to $\overline{RS}$, we will use the right angle formed to construct a square.

5. Label the midpoint of $\overline{RS}$ on the diagram as point M. Using $\overline{MS}$ as one side of the square, and the right angle formed by $\overline{SM}$ and the perpendicular line drawn through point M as the beginning of a square, finish constructing this square on the diagram. (Hint: Remember that a square is also a rhombus, and you have already constructed a rhombus in the first part of this task.)

Ready for More?

Draw a line and select two arbitrary points on the line. Treating the line segment between the two points you selected as one side of a square, use the construction strategies you invented in today's task to construct the square. Demonstrate your thinking by showing all of the circles, or portions of circles, you use in your construction.

Takeaways

I used circles and lines as a construction tool today.

- Circles are useful construction tools because _____
- Congruent circles are useful construction tools because _____
- The congruent circles in the construction of the perpendicular bisector helped me to notice that _____

Vocabulary

- congruence statement
- **construction**
- equality statements
- radii
- ray

Bold terms are new in this lesson.

Lesson Summary

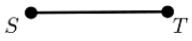
In this lesson, we learned about constructions: creating geometric figures precisely, using only a compass and a straightedge. Using only these tools, we constructed a rhombus with a given side and angle, constructed the perpendicular bisector of a side, and constructed a square with a given right angle and line segment for a side. We learned the value of the definition of a circle—*the set of all points in a plane equidistant from a fixed center point*—since circles allow us to construct congruent line segments.

Unit 7, Lesson 2

More Things Under Construction

Jump Start

Like a rhombus, an equilateral triangle has congruent sides. Show and describe how you might locate the third vertex point on an equilateral triangle, given $\overline{ST}$ as one side of the equilateral triangle.



Learning Focus

Construct parallel lines and inscribed regular polygons.

How do I use geometric objects, such as circles and lines, to construct geometric figures like parallel lines and regular polygons, rather than using measurement tools, such as rulers and protractors, to draw such figures?

Open Up the Math: Launch, Explore, Discuss

Because **regular polygons** have rotational symmetry, they can be inscribed in a circle. The circumscribed circle has its center at the center of rotation and passes through all of the vertices of the regular polygon.

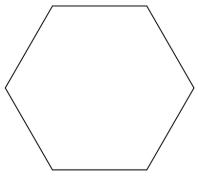
We might begin constructing a hexagon by noticing that a hexagon can be decomposed into six congruent equilateral triangles, formed by three of its lines of symmetry.

1.

   a. Sketch a diagram of such a decomposition.

   b. Based on your sketch, where is the center of the circle that would circumscribe the hexagon?

   c. Use a compass to draw the circle that would circumscribe the hexagon.



Constructing a Regular Hexagon Inscribed in a Circle

2. The six vertices of the regular hexagon lie on the circle in which the regular hexagon is inscribed. The six sides of the hexagon are **chords** of the circle. How are the lengths of these chords related to the lengths of the radii from the center of the circle to the vertices of the hexagon? That is, how do you know that the six triangles formed by drawing the three lines of symmetry are equilateral triangles? (Hint: Considering angles of rotation, can you convince yourself that these six triangles are equiangular and therefore equilateral?)

3. Based on this analysis of the regular hexagon and its circumscribed circle, illustrate and describe a process for constructing a hexagon inscribed in the given circle.



4. Modify your work with the hexagon to construct an equilateral triangle inscribed in the given circle.



Constructing a Parallel Line Through a Given Point

5. It is often useful to be able to construct a line parallel to a given line through a given point. For example, suppose we want to construct a line parallel to $\overline{AB}$ through point C on the diagram below. Since we have observed that parallel lines have the same slope, the line through point C will be parallel to $\overline{AB}$ only if the angle formed by the line and $\overrightarrow{BC}$ is congruent to $\angle ABC$. Can you describe and illustrate a strategy that will construct an angle with the vertex at point C and a side parallel to $\overline{AB}$?



Ready for More?

Describe how you might construct a square inscribed in a circle. Test out your construction on the circle below.



Takeaways

Strategies for constructing geometric figures:

Vocabulary

- **circumscribe**
- **inscribed in a circle**

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we learned how to construct regular hexagons and equilateral triangles by inscribing them in a circle. We also learned how to construct a line parallel to a given line through a given point. These constructions were based on properties of figures that we have observed in previous lessons.

Unit 7, Lesson 3

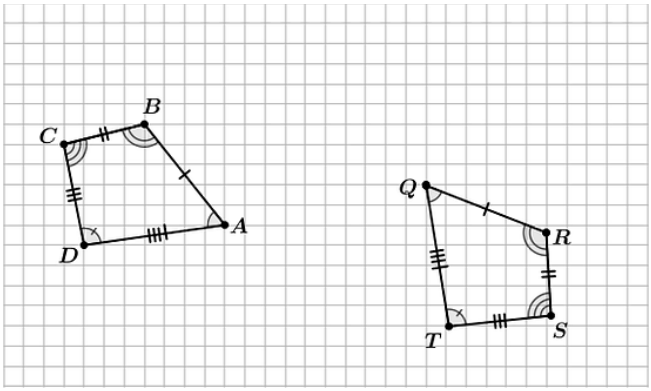
Can You Get There From Here?

Jump Start

Notice and Wonder

Record at least two things you notice and one thing you wonder about the diagram and description given:

Given: $ABCD \cong QRST$



What I noticed:	What I am wondering:

Learning Focus

Show two figures are congruent based on an efficient and consistent sequence of rigid transformations.

How do I know if two images, such as the frog and lizard images of previous tasks, are congruent?

Is there a "best" sequence of transformations for showing that two figures are congruent to each other? What features of the figures themselves support this work?

Is there a sequence of transformations that would by easy to replicate every time?

Open Up the Math: Launch, Explore, Discuss

The two quadrilaterals shown, quadrilateral $ABCD$ and quadrilateral $QRST$, are congruent, with corresponding congruent parts marked in the diagrams.

Describe a sequence of rigid-motion transformations that will carry quadrilateral $ABCD$ onto quadrilateral $QRST$. Be very specific in describing the sequence and types of transformations you will use so that someone else could perform the same series of transformations.

Once you have a written description, exchange it with another student or two and see if each of you can follow the other's strategy to show that the two figures are congruent. You should ask questions of each other and suggest edits that will help each of you refine your descriptions.

Here are some questions for reflection that will help:

• Look carefully at the words and diagrams. Do they make sense?

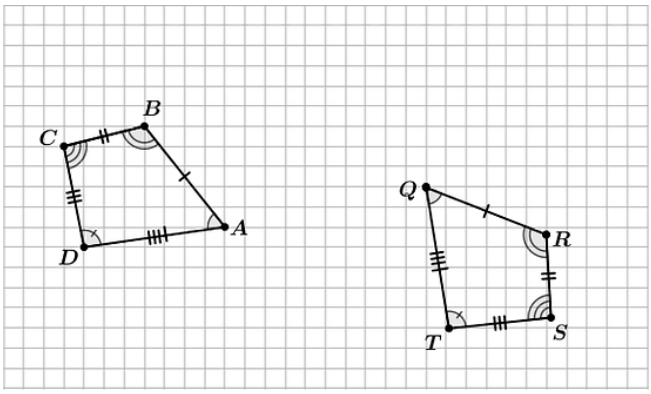
• Do we see a mistake that needs to be fixed?

• Did we leave something out that needs to be put in?

• Do we know something more that needs to be added?

• Why do we want to make the changes we are proposing?

1. Complete:



Ready for More?

In the Ready for More? for Unit 6 Lesson 4, we learned that reflecting an image consecutively over two intersecting lines produces a rotation. (Try this out if you didn't do the Ready for More? in Unit 6 Lesson 4.)

1. What happens if we reflect an image consecutively over two parallel lines? (Try out several examples until you can explain what happens when an image is reflected consecutively over two parallel lines.)

2. Based on these observations about what happens when a figure is reflected consecutively over two intersecting lines, or over two parallel lines, can you carry one of two congruent figures onto the other using only reflections? Do you think this is always possible? Why do you think so?

Takeaways

Congruent figures are the same size and shape.

To prove that two figures are congruent, I need to _____.

A good strategy for doing this is:

Once I have shown that two figures are congruent, I know that corresponding segments and angles in the two figures are congruent because _____.

Vocabulary

- corresponding angles
- corresponding points / sides

Lesson Summary

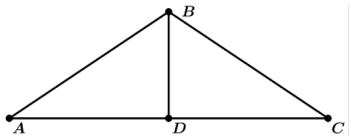
In this lesson, we explored a sequence of rigid transformations that could be used to demonstrate that one geometric figure is congruent to another. While many sequences can be found, one particular sequence was identified as being more consistent and easy to replicate each time we need to show that two figures are congruent.

Unit 7, Lesson 4

Congruent Triangles

Jump Start

Examine the diagram.



Is $\triangle ABD$ congruent to triangle $\triangle CBD$? List all of the things you would need to know in order to know for sure that the two triangles are congruent.

Learning Focus

Explore and justify triangle congruence criteria using rigid transformations.

What do I need to know about two triangles before I can say that they are congruent?

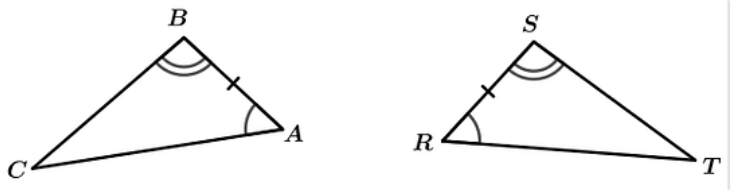
How do I verify that a set of criteria that seems to imply triangles are congruent will always work?

Open Up the Math: Launch, Explore, Discuss

We know that two triangles are congruent if all pairs of corresponding sides are congruent and all pairs of corresponding angles are congruent. We may wonder if knowing less information about the triangles would still guarantee that they are congruent.

For example, we may wonder if knowing that two angles and the included side of one triangle are congruent to the corresponding two angles and the included side of another triangle—a set of criteria we will refer to as ASA—is enough to know that the two triangles are congruent. And, if we think that this is enough information, how might we justify that this is true?

Here is a diagram illustrating ASA criteria for triangles:



1. Based on the diagram, which angles are congruent? Which sides are congruent?
2. To convince ourselves that these two triangles are congruent, what else would we need to know?
3. Use tracing paper to find a sequence of transformations that will show whether or not these two triangles are congruent.
4. List your sequence of transformations:

Your sequence of transformations is enough to show that these two triangles are congruent, but how can we guarantee that all pairs of triangles that share ASA criteria are congruent?

Perhaps your sequence of transformations looked like this:

- **translate** point A until it coincides with point R.
- **rotate** $\overline{AB}$ about point R until it coincides with $\overline{RS}$.
- **reflect** $\triangle ABC$ across $\overleftrightarrow{RS}$.

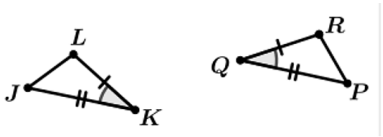
We can use the word ***coincides*** when we want to say that two points or line segments occupy the same position on the plane. When making arguments using transformations, we will use the word a lot.

Now the question is, how do we know that point C has to land on point T after the reflection, making all of the sides and angles coincide?

5. Answer this problem as best you can to justify why ASA criteria guarantee that two triangles are congruent. It may be helpful to think about how you know point C can't land anywhere else in the plane except on top of T.

Using tracing paper, experiment with these additional pairs of triangles. Try to determine if you can find a sequence of transformations that will show whether the triangles are congruent. Be careful; there may be some that aren't. If the triangles appear to be congruent based on your experimentation, write an argument to explain how you know that this type of criterion will always work. That is, what guarantees that the unmarked sides or angles must also coincide?

6. Complete:



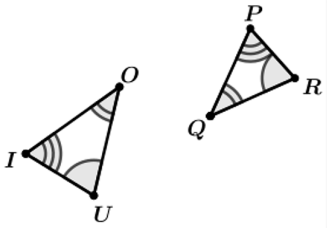
Given criterion:

Are the triangles congruent?

List your transformations in the order performed:

If the triangles are congruent, justify why this will always be true based on this criterion:

7. Complete:



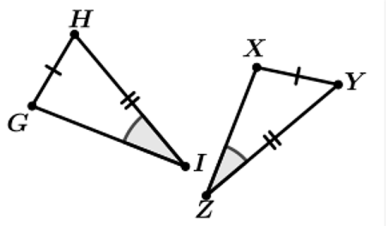
Given criterion:

Are the triangles congruent?

List your transformations in the order performed:

If the triangles are congruent, justify why this will always be true based on this criterion:

8. Complete:



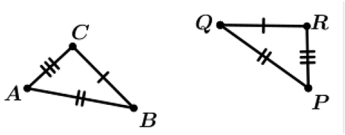
Given criterion:

Are the triangles congruent?

List your transformations in the order performed:

If the triangles are congruent, justify why this will always be true based on this criterion:

9. Complete:



Given criterion:

Are the triangles congruent?

List your transformations in the order performed:

If the triangles are congruent, justify why this will always be true based on this criterion:

10. Based on these experiments and your justifications, what criteria or conditions seem to guarantee that two triangles will be congruent? List as many examples as you can. Make sure you include ASA from the triangles we worked with first.

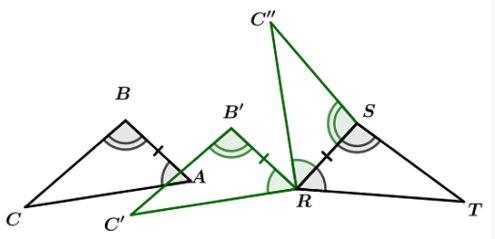
11. Your friend wants to add AAS to your list, even though you haven't experimented with this particular case. What do you think? Should AAS be added or not? What convinces you that you are correct?

12. Your friend also wants to add HL (hypotenuse-leg) to your list, even though you haven't experimented with right triangles at all, and you know that SSA doesn't work in general from problem 8. What do you think? Should HL for right triangles be added or not? What convinces you that you are correct?

Ready for More?

Sione and Zac are working on their precision in language as they critique each other's thinking on the first problem of this lesson.

Sione has been watching Zac experiment with the following pair of triangles that have three corresponding parts which are congruent, and he has a concern. "How do we know the conditions you found that make congruent triangles in your experiments will work for all triangles, and not just for these triangles?"



Zac's argument:

"I know what I did for the first set of triangles," says Zac, "and I think I can do the same for any pair of triangles that have ASA criteria marked."

"We can translate point A until it coincides with point R, then rotate $\overline{AB}$ about point R until it coincides with $\overline{RS}$. Finally, we can reflect $\triangle ABC$ across $\overleftrightarrow{RS}$, and then everything coincides so that the triangles are congruent." (Zac and Sione's teacher has suggested they use the word *coincides* when they want to say that two points or line segments occupy the same position on the plane. They like the word, so they plan to use it a lot.)

What do you think about Zac's argument? Does it convince you that the two triangles are congruent? Does it leave out any essential ideas that you think need to be included? Reflect on these questions as you read Sione and Zac's discussion.

Sione isn't sure that Zac's argument is really convincing. He asks Zac, "How do you know point C coincides with point T after you reflect the triangle?"

1. How do you think Zac might answer Sione's question?

While Zac is trying to think of an answer to Sione's question, he adds this comment, "And you really didn't use all of the information about the corresponding congruent parts of the two triangles."

"What do you mean?" asks Zac.

Sione replies, "You started using the fact that $\angle A \cong \angle R$ when you translated $\triangle ABC$, so that vertex A coincides with vertex R. And you used the fact that $\overline{AB} \cong \overline{RS}$ when you rotated $\overline{AB}$ to coincide with $\overline{RS}$, but where did you use the fact that $\angle B \cong \angle S$?"

"Yeah, and what does it really mean to say that two angles are congruent?" Zac adds. "Angles are more than just their vertex points."

2. How might thinking about Zac and Sione's questions help improve Zac's argument?

Sione's argument:

"I would start the same way you did, by translating point A until it coincides with point R, rotating $\overline{AB}$ about point R until it coincides with $\overline{RS}$, and then reflecting $\triangle ABC$ across $\overline{RS}$," Sione says. "But then I would want to convince myself that points C and T coincide. I know that an angle is made up of two rays that share a common endpoint. Since I know that $\overrightarrow{AB}$ coincides with $\overrightarrow{RS}$ and $\angle A \cong \angle R$, that means that $\overrightarrow{AC}$ coincides with $\overrightarrow{RT}$. Likewise, I know that $\overrightarrow{BA}$ coincides with $\overrightarrow{SR}$ and $\angle B \cong \angle S$, so $\overrightarrow{BC}$ must coincide with $\overrightarrow{ST}$. Since $\overrightarrow{AC}$ and $\overrightarrow{BC}$ intersect at point C, and $\overrightarrow{RT}$ and $\overrightarrow{ST}$ intersect at point T, points C and T must also coincide, because the corresponding rays coincide. Therefore, $\overline{BC} \cong \overline{ST}$, $\overline{CA} \cong \overline{TR}$, and $\angle C \cong \angle T$, because both angles are made up of rays that coincide!"

At first, Zac is confused by Sione's argument, but he draws diagrams and carefully marks and sketches out each of his statements until it starts to slowly make sense.

3. Do the same kind of work that Zac did to make sense of Sione's argument. What ideas did sketching out the words of his proof help you to clarify? What parts of his argument are unclear to you?

Takeaways

When working with transformations, we use words like *coincide*, *superimposed*, or *carried onto* to refer to

_____

One way we can justify a claim is to use a proof by contradiction method, in which

_____

Sufficient criteria to guarantee that two triangles are congruent:

Criteria that may not guarantee that triangles are congruent (along with observations about what may happen under these conditions):

Adding Notation, Vocabulary, and Conventions

We use the following notation to indicate that corresponding parts of two triangles are congruent.

Given two triangles:

SAS (side-angle-side) means: _____

ASA (angle-side-angle) means: _____

SSS (side-side-side) means: _____

Vocabulary

- **coincides (superimposed or carried onto)**
- **corresponding parts (in a triangle)**
- **counterexample**
- **proof by contradiction**
- **triangle congruence criteria: ASA, SAS, AAS, SSS**

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we learned that it is not necessary to know that all pairs of corresponding angles and sides are congruent before we can claim that two triangles are congruent. There are several conditions where three pieces of information about corresponding congruent parts of the two triangles are sufficient to guarantee congruence of the triangles. We were able to justify the triangle congruence criteria by relying on the properties of rigid transformations to preserve distance and angle measures.

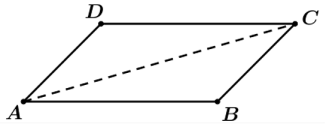
Unit 7, Lesson 5

Congruent Triangles to the Rescue

Jump Start

Previously, we have conjectured that the opposite sides of a parallelogram are congruent, based on experimentation and an explanation using rigid transformations.

1. Based on this assumption, mark congruent segments in the diagram.



2. Would drawing one of the diagonals of the parallelogram provided in problem 1 divide the parallelogram into two congruent triangles?

3. If so, what triangle congruence criterion did you use to decide that the two triangles are congruent?

4. Name a pair of congruent angles in the diagram provided in problem 1.

5. How do you know those angles are congruent?

Learning Focus

Identify congruent triangles, and write congruency statements.

Use triangle congruence criteria to justify other properties of geometric figures.

How can I capture what I see when I identify congruent triangles in symbolic notation?

How do I use the triangle congruence criteria to justify other properties of geometric figures?

When might it be helpful to decompose a geometric figure into triangles so that I can use triangle congruence criteria to prove something else about the figure?

Open Up the Math: Launch, Explore, Discuss

Part 1

Zac and Sione are exploring isosceles triangles—triangles in which two sides are congruent:

Zac: "I think every isosceles triangle has a line of symmetry that passes through the vertex point of the angle made up by the two congruent sides and the midpoint of the third side."

Sione: "That's a pretty big claim—to say you know something about every isosceles triangle. Maybe you just haven't thought about the ones for which the claim isn't true."

Zac: "But I've folded lots of isosceles triangles in half, and it always seems to work."

Sione: "Lots of isosceles triangles are not all isosceles triangles, so I'm still not sure."

1. What do you think about Zac's claim? Do you think every isosceles triangle has a line of symmetry? If so, what convinces you this is true? If not, what concerns do you have about his statement?

2. What else would Zac need to know about the crease line in order to know that it is a line of symmetry? (Hint: Think about the definition of a line of reflection.)

3. Sione thinks Zac's "crease line" (the line formed by folding the isosceles triangle in half) creates two congruent triangles inside the isosceles triangle. Which criterion—ASA, SAS, or SSS—could he use to support this claim? Describe the sides and/or angles you think are congruent, and explain how you know they are congruent.

4. If the two triangles created by folding an isosceles triangle in half are congruent, what does that imply about the "base angles" of an isosceles triangle (the two angles that are not formed by the two congruent sides)?

5. If the two triangles created by folding an isosceles triangle in half are congruent, what does that imply about the "crease line"? (You might be able to make a couple of claims about this line—one claim comes from focusing on where the line meets the third, noncongruent side of the triangle; a second claim comes from focusing on where the line intersects with the vertex angle formed by the two congruent sides.)

Part 2

Like Zac, you have done some experimenting with lines of symmetry, as well as with rotational symmetry. In the tasks you did in *Symmetries of Quadrilaterals* and *Quadrilaterals—Beyond Definition,* you made some observations about sides, angles, and diagonals of various types of quadrilaterals, based on your experiments and knowledge about transformations. Many of these observations can be further justified based on looking for congruent triangles and their corresponding parts, just as Zac and Sione did in their work with isosceles triangles.

Pick one of the following quadrilaterals to explore:

- A **rhombus** is a quadrilateral in which all sides are congruent.
- A **rectangle** is a quadrilateral that contains four right angles.
- A **square** is both a rectangle and a rhombus, that is, it contains four right angles and all sides are congruent.

6. Draw an example of your selected quadrilateral with its diagonals. Label the vertices of the quadrilateral A, B, C, and D, and label the point of intersection of the two diagonals as point N.

7. Based on (1) your drawing, (2) the given definition of your quadrilateral, and (3) information about sides and angles that you can gather based on lines of reflection and rotational symmetry, list as many pairs of congruent triangles as you can find.

8. For each pair of congruent triangles that you list, state the criterion you used—ASA, SAS, or SSS–to determine that the two triangles are congruent, and explain how you know that the angles and/or sides required by the criterion are congruent (see the chart).

Congruent Triangles	Criteria Used (ASA, SAS, SSS)	How do I know that the sides and/or angles required by the criteria are congruent?
Example: If I say $\triangle RST \cong \triangle XYZ$	Based on SSS	Then I need to explain: – how I know that $\overline{RS} \cong \overline{XY}$, and – how I know that $\overline{ST} \cong \overline{YZ}$, and – how I know that $\overline{TR} \cong \overline{ZX}$ so I can use SSS criteria to say $\triangle RST \cong \triangle XYZ$.

Ready for More?

We have accepted many attributes of different types of parallelograms based on experimentation with rigid transformations. Now that you have identified some congruent triangles in parallelograms, try to use the congruent triangles to justify something else about the different types of parallelograms, such as:

- The diagonals bisect each other.
- The diagonals are congruent.
- The diagonals are perpendicular to each other.
- The diagonals bisect the angles of the quadrilateral.

Pick one of the bulleted statements you think is true about a specific type of parallelogram, and try to write an argument that would convince Zac and Sione that the statement is true.

Takeaways

Today I learned several strategies for working with diagrams to prove statements about congruence, such as:

Adding Notation, Vocabulary, and Conventions

Sometimes it is useful to add an auxiliary line to a diagram.

An auxiliary line is _____

In this case, the auxiliary line allowed me to _____

Vocabulary

- **CPCTC**
- **auxiliary line**

Bold terms are new in this lesson.

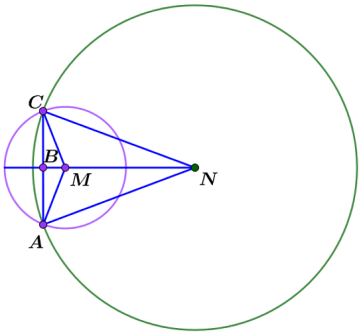
Lesson Summary

In this lesson, we found several strategies for justifying statements about congruence by first looking for congruent triangles within a geometric figure. One helpful strategy was adding auxiliary lines to the figure. Another was to use the idea that corresponding parts of congruent triangles are congruent.

Unit 7, Lesson 6
Justifying Constructions

Jump Start

We examined this diagram in a previous Jump Start and wondered if we could claim if the two segments, $\overline{AB}$ and $\overline{CB}$, were congruent.



How can you use triangle congruence criteria, such as ASA, SAS, or SSS, to prove that these two segments are congruent? (If you need help, your teacher has some hints to get you started.)

Learning Focus

Justify construction strategies.

How can I explain why particular constructions (made only with lines and circular arcs) work?

How might I draw upon triangle congruence criteria and the definitions of rigid transformations in my explanations?

Open Up the Math: Launch, Explore, Discuss

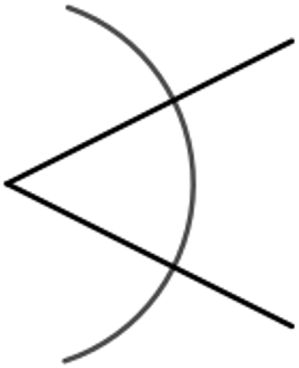
Compass and straightedge constructions can be justified using such tools as:

• definitions and properties of the rigid transformations

• identifications of the corresponding parts of congruent triangles

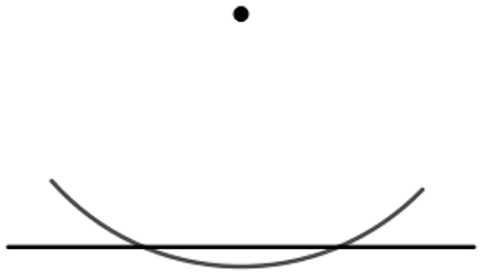
• conjectures about the attributes of sides, angles, and diagonals of special types of quadrilaterals

1. Study the steps of the following procedure for constructing an angle bisector, and complete the illustration based on the descriptions of the steps.

	Steps	Illustration
	Using a compass, draw an arc (portion of a circle) that intersects each ray of the angle to be bisected, with the center of the arc located at the vertex of the angle.	
	Without changing the span of the compass, draw two arcs in the interior of the angle, with the center of the arcs located at the two points where the first arc intersected the rays of the angle.	
	With the straightedge, draw a ray from the vertex of the angle through the point where the last two arcs intersect.	

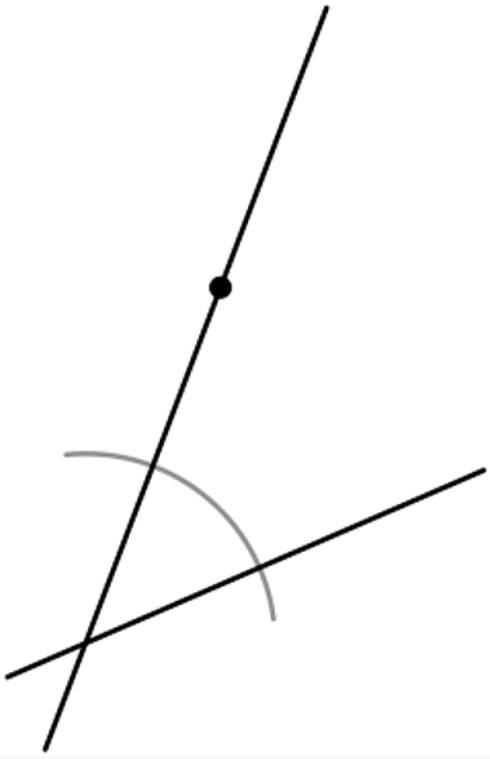
Explain in detail why this construction works. It may be helpful to identify some congruent triangles or a familiar quadrilateral in the final illustration. You may also want to use definitions or properties of the rigid-motion transformations in your explanation. Be prepared to share your explanation with your peers.

2. Study the steps of the following procedure for constructing a line perpendicular to a given line through a given point, and complete the illustration based on the descriptions of the steps.

Steps	Illustration
Using a compass, draw an arc (portion of a circle) that intersects the given line at two points, with the center of the arc located at the given point.	
Without changing the span of the compass, locate a second point on the other side of the given line by drawing two arcs on the same side of the line, with the center of the arcs located at the two points where the first arc intersected the line.	
With the straightedge, draw a line through the given point and the point where the last two arcs intersect.	

Explain in detail why this construction works. It may be helpful to identify some congruent triangles or a familiar quadrilateral in the final illustration. You may also want to use definitions or properties of the rigid-motion transformations in your explanation. Be prepared to share your explanation with your peers.

3. Study the steps of the following procedure for constructing a line parallel to a given line through a given point, and complete the illustration based on the descriptions of the steps.

Steps	Illustration
Using a straightedge, draw a line through the given point to form an arbitrary angle with the given line.	
Using a compass, draw an arc (portion of a circle) that intersects both rays of the angle formed, with the center of the arc located at the point where the drawn line intersects the given line.	
Without changing the span of the compass, draw a second arc on the same side of the drawn line, centered at the given point. The second arc should be as long or longer than the first arc and should intersect the drawn line.	
Set the span of the compass to match the distance between the two points where the first arc crosses the two lines. Without changing the span of the compass, draw a third arc that intersects the second arc, centered at the point where the second arc intersects the drawn line.	
With the straightedge, draw a line through the given point and the point where the last two arcs intersect.	

Explain in detail why this construction works. It may be helpful to identify some congruent triangles or a familiar quadrilateral in the final illustration. You may also want to use definitions or properties of the rigid-motion transformations in your explanation. Be prepared to share your explanation with your peers.

Ready for More?

Draw a line segment and a point not on the line. Using the line segment as one side of a parallelogram and the point as one of the vertices of the parallelogram, construct the parallelogram $ABCD$ with a segment h representing the height of the parallelogram constructed from one vertex perpendicular to the opposite side.

Takeaways

Strategies I might use when justifying constructions:

- _____
- _____
- _____

These strategies will also be useful in future geometry work.

Lesson Summary

In this lesson, we examined some standard constructions, such as bisecting an angle, constructing a line perpendicular to a given line through a given point, and constructing a line parallel to a given line through a given point. We found that we could explain why these constructions work based on properties of quadrilaterals, using corresponding parts of congruent triangles, or relying on the defining features and properties of rigid transformations.