Unit 6, Lesson 1

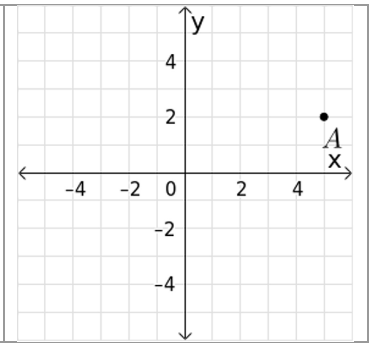
Leaping Lizards!

Jump Start

Complete:

Given the **pre-image** point A located at $(5,2)$ as shown in the diagram, find the location of the **image** point if point A is:

a. Reflected over the x-axis. _____
b. Reflected over the y-axis. _____
c. Rotated $180°$ counterclockwise around the origin. _____
d. Rotated $90°$ clockwise around the origin. _____
e. Translated left 2 units and up 3 units. _____



Learning Focus

Identify features of translations, rotations, and reflections.

What tools and strategies do I use when I translate a figure? Rotate a figure? Reflect a figure?

What do these tools and strategies reveal about the transformation?

Although each of these transformations move figures in different ways, what do they all have in common?

Open Up the Math: Launch, Explore, Discuss

Animated films and cartoons are now usually produced using computer technology, rather than the hand-drawn images used in the past. Computer animation requires both artistic talent and mathematical knowledge.

Sometimes animators want to move an image around the computer screen without distorting the size and shape of the image in any way. This is done using **geometric transformations** such as translations (slides), reflections (flips), and rotations (turns), or perhaps some combination of the three. These transformations need to be precisely defined, so there is no doubt about where the final image will end up on the screen.

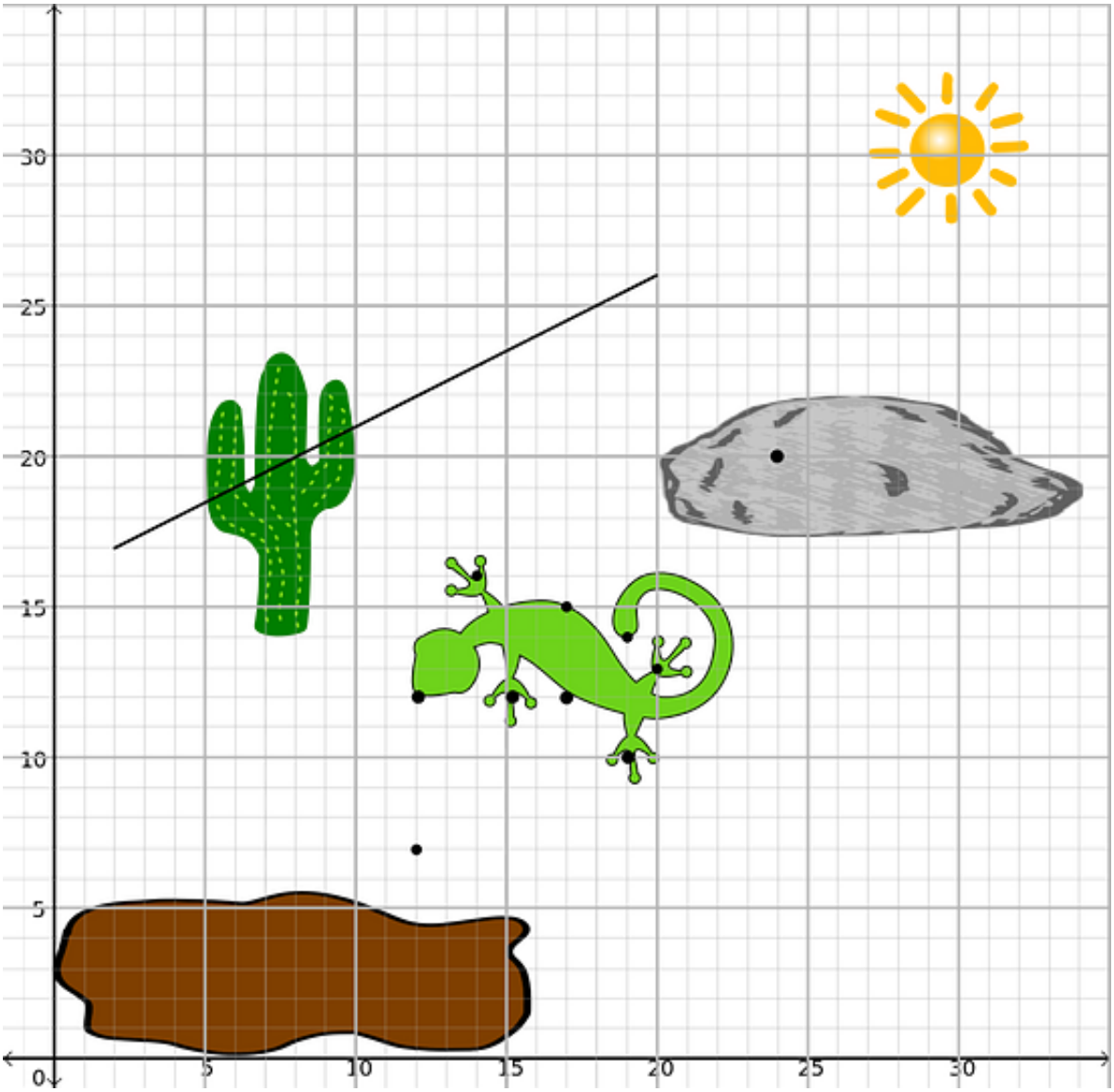
Today you will be animating the lizard shown on your handout by transforming it in various ways. The original lizard was created by plotting the following anchor points on the coordinate grid and then letting a computer program draw the lizard. The anchor points are always listed in this order: tip of nose, center of left front foot, belly, center of left rear foot, point of tail, center of rear right foot, back, center of front right foot.

Original lizard anchor points:

$\{(12,12), (15,12), (17,12), (19,10), (19,14), (20,13), (17,15), (14,16)\}$

Each statement below describes a transformation of the original lizard. Do the following for each of the transformations:

- Plot the anchor points for the lizard in its new location.
- Connect the pre-image and image anchor points with line segments or circular arcs, whichever best illustrates the relationship between them.



1. **Lazy Lizard**

   Translate the original lizard so the point at the tip of its nose is located at $(24,20)$, making the lizard appear to be sunbathing on the rock.

2. **Lunging Lizard**

   Rotate the lizard $90°$ about the point $A(12,7)$ so it looks like the lizard is diving into the puddle of mud.

3. **Leaping Lizard**

   Reflect the lizard about the given line $y = \frac{1}{2}x + 16$ so it looks like the lizard is doing a back flip over the cactus.

Ready for More?

Refer to Lazy Lizard, the lizard lounging on the rock. Lazy Lizard is rotated $90°$ counterclockwise about the point $(12,7)$. Find the coordinates of the points at the tip of the nose and the end of the tail for the rotated image of the lizard.

Describe how you found these points since they do not lie on the given portion of the coordinate grid.

Takeaways

What we noticed about all three transformations:

What we noticed about each transformation:

- Translations: _____
- Rotations: _____
- Reflections: _____

Adding Notation, Vocabulary, and Conventions

Use this notation to describe a translation: _____

Vocabulary

- clockwise / counterclockwise
- **congruent (CPCTC)**
- **image**
- **orientation**
- origin
- **pre-image / image**
- **reflection**
- **rigid transformation**
- **rotation**
- **translation**

Bold terms are new in this lesson.

Lesson Summary

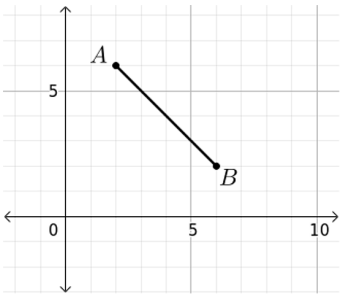
In this lesson, we explored how to perform rigid transformations using a variety of tools, such as tracing paper, rulers, protractors, and compasses; and using a variety of methods, such as counting the squares on the coordinate grid, drawing parallel lines, or folding an image over a line. We used these tools and strategies to identify key features of each of the transformations.

Unit 6, Lesson 2

Is It Right?

Jump Start

The given line segment in the coordinate grid, with endpoints at $A(2,6)$ and $B(6,2)$, is one side of a triangle. Describe the triangle formed if each of the following points is the third vertex C of the triangle. (Use words in your description such as acute, obtuse, right, scalene, isosceles, equilateral.)



1. Complete:

   If $C(2,4)$ is the third vertex of the triangle then the triangle is _____.

2. Complete:

   If $C(2,2)$ is the third vertex of the triangle then the triangle is _____.

3. Complete:

   If $C(0,0)$ is the third vertex of the triangle then the triangle is _____.

4. Complete:

   If $C(6,6)$ is the third vertex of the triangle then the triangle is _____.

5. Complete:

   If $C(3,2)$ is the third vertex of the triangle then the triangle is _____.

6. Complete:

   If $C(1,2)$ is the third vertex of the triangle then the triangle is _____.

Learning Focus

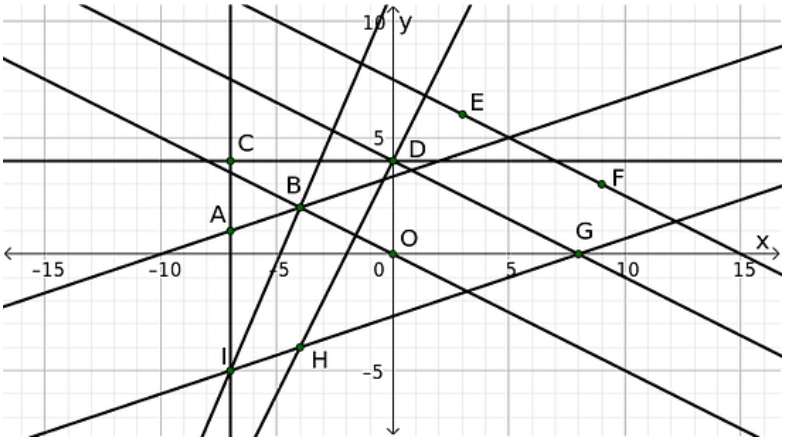
Recognize parallel and perpendicular lines in a coordinate plane.

How can I determine if two lines in a coordinate plane are parallel or perpendicular? Is visual examination (that is, “It looks like it”) sufficient?

Open Up the Math: Launch, Explore, Discuss

In *Leaping Lizards,* you probably thought a lot about parallel and perpendicular lines, particularly when you translated the anchor points of the lizard the same distance and in the same direction, or rotated the lizard about a given center through a 90° angle, or reflected the lizard across a line. It would be helpful to be able to predict when lines in a coordinate grid are parallel or perpendicular to each other.

1. Examine the following diagram and write statements to describe which lines are parallel to each other.



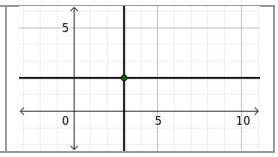
My list of parallel lines:

How did you know that these lines were parallel, other than, "They look like parallel lines?"

You may have written your statements about parallel lines using words such as, "The line through the points A and B is parallel to the line through the points H and G." We can state this same idea symbolically.

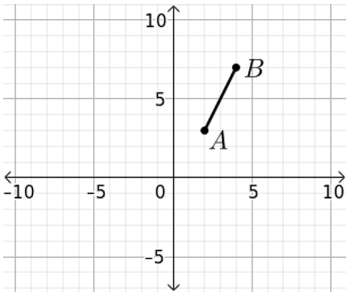
Now that we have made an observation about the slopes of parallel lines, it will be helpful to make an observation about the slopes of perpendicular lines. Perhaps in *Leaping Lizards* you used a protractor or some other tool or strategy to help you make a right angle. In this task we consider how to create a right angle by attending to slopes on the coordinate grid.

We begin by stating a fundamental idea for our work: Horizontal and vertical lines are perpendicular. For example, on a coordinate grid, the horizontal line $y = 2$ and the vertical line $x = 3$ intersect to form four right angles.



But what if a line or line segment is not horizontal or vertical? How do we determine the slope of a line or line segment that will be perpendicular to it?

2. **Experiment 1**



a. Consider the points $A(2,3)$ and $B(4,7)$ and the line segment, $\overline{AB}$, between them. What is the slope of this line segment?

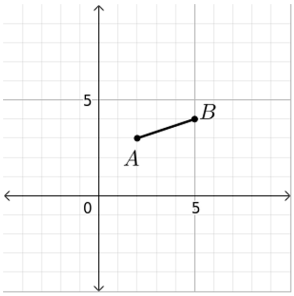
b. Locate and name a third point $C(x,y)$ on the coordinate grid, so the points $A(2,3)$,$B(4,7)$ and $C(x,y)$ form the vertices of a right triangle, with $\overline{AB}$ as its hypotenuse.

c. Explain how you know that the triangle you formed contains a right angle.

d. Now rotate this right triangle $90°$ about the vertex point $A(2,3)$. Explain how you know that you have rotated the triangle $90°$.

e. Compare the slope of the hypotenuse of this rotated right triangle with the slope of the hypotenuse of the pre-image. What do you notice?

3. **Experiment 2**



a. Consider the points $A(2,3)$ and $B(5,4)$ and the line segment, $\overline{AB}$, between them. What is the slope of this line segment?

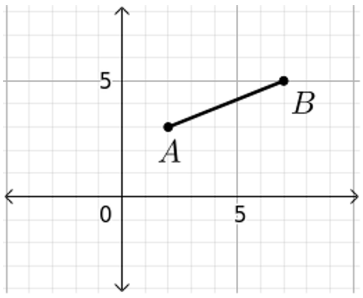
b. Locate and name a third point $C(x,y)$ on the coordinate grid, so the points $A(2,3)$,$B(5,4)$ and $C(x,y)$ form the vertices of a right triangle, with $\overline{AB}$ as its hypotenuse.

c. Explain how you know that the triangle you formed contains a right angle.

d. Now rotate this right triangle $90°$ about the vertex point $A(2,3)$. Explain how you know that you have rotated the triangle $90°$.

e. Compare the slope of the hypotenuse of this rotated right triangle with the slope of the hypotenuse of the pre-image. What do you notice?

4. **Experiment 3**



a. Consider the points $A(2,3)$ and $B(7,5)$ and the line segment, $\overline{AB}$, between them. What is the slope of this line segment?

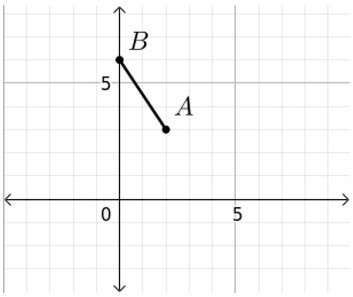
b. Locate a third point $C(x, y)$ on the coordinate grid, so the points $A(2,3)$, $B(7,5)$, and $C(x, y)$ form the vertices of a right triangle, with $\overline{AB}$ as its hypotenuse.

c. Explain how you know that the triangle you formed contains a right angle.

d. Now rotate this right triangle $90°$ about the vertex point $A(2,3)$. Explain how you know that you have rotated the triangle $90°$.

e. Compare the slope of the hypotenuse of this rotated right triangle with the slope of the hypotenuse of the pre-image. What do you notice?

5. **Experiment 4**



a. Consider the points $A(2,3)$ and $B(0,6)$ and the line segment, $\overline{AB}$, between them. What is the slope of this line segment?

b. Locate a third point $C(x, y)$ on the coordinate grid, so the points $A(2,3)$, $B(0,6)$, and $C(x, y)$ form the vertices of a right triangle, with $\overline{AB}$ as its hypotenuse.

c. Explain how you know that the triangle you formed contains a right angle.

d. Now rotate this right triangle $90°$ about the vertex point $A(2,3)$. Explain how you know that you have rotated the triangle $90°$.

e. Compare the slope of the hypotenuse of this rotated right triangle with the slope of the hypotenuse of the pre-image. What do you notice?

6. Based on experiments 1–4, state an observation about the slopes of perpendicular lines.

   While this observation is based on a few specific examples, can you create an argument or justification for why this is always true?

Now that we know how to identify perpendicular lines on a coordinate grid, we can also use symbolic notation to indicate that the two lines are perpendicular.

Ready for More?

For experiments 1–4, write the equations of the lines that contain the hypotenuse of the original triangle and the hypotenuse of the rotated triangle in both point-slope form and slope-intercept form, using the point at the center of rotation for the point-slope form of the equation. What relationships do you notice in these equations?

Takeaways

When working with lines on a coordinate grid,

- I know the lines are parallel if _____
- I know the lines are perpendicular if _____

Adding Notation, Vocabulary, and Conventions

I can indicate two lines are parallel using the following notation:

In representing parallel lines on a coordinate grid, we have drawn images to represent the following undefined terms: point, line, and plane. These are abstract ideas, rather than concrete objects because:

- Unlike a dot, a point _____
- Unlike a straight stroke of ink, a line _____
- Unlike a piece of paper, a plane _____

Vocabulary

- Pythagorean theorem
- acute angle
- equilateral, equilateral triangle
- isosceles triangle, trapezoid
- line
- line segment
- obtuse angle / obtuse triangle
- **opposite (or negative) reciprocal slope**
- **parallel line**
- **perpendicular lines**
- plane
- point
- right angle
- scalene triangle

Lesson Summary

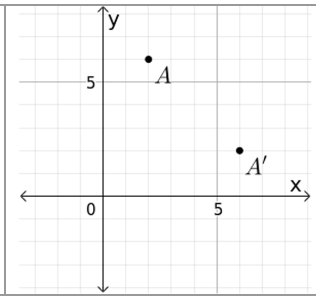
In this lesson, we learned criteria for determining if two lines in a coordinate plane are parallel or perpendicular. We also learned notation for indicating that lines are parallel or perpendicular in our written work.

Unit 6, Lesson 3

Leap Frog

Jump Start

Given the pre-image point A located at $(2,6)$ and the image point A' located at $(6,2)$ as shown in the diagram, describe the following transformations:

A translation that will carry A onto A' A reflection that will carry A onto A' A rotation that will carry A onto A' A sequence of two transformations that will carry A onto A'	

Learning Focus

Determine the rigid transformation that carries one image onto another.

When given an image and its pre-image, what do I look for to identify a rigid transformation that has already occurred?

How do I recognize when a sequence of rigid transformations may be needed to carry one figure onto another, and how do I identify what transformations that sequence might include?

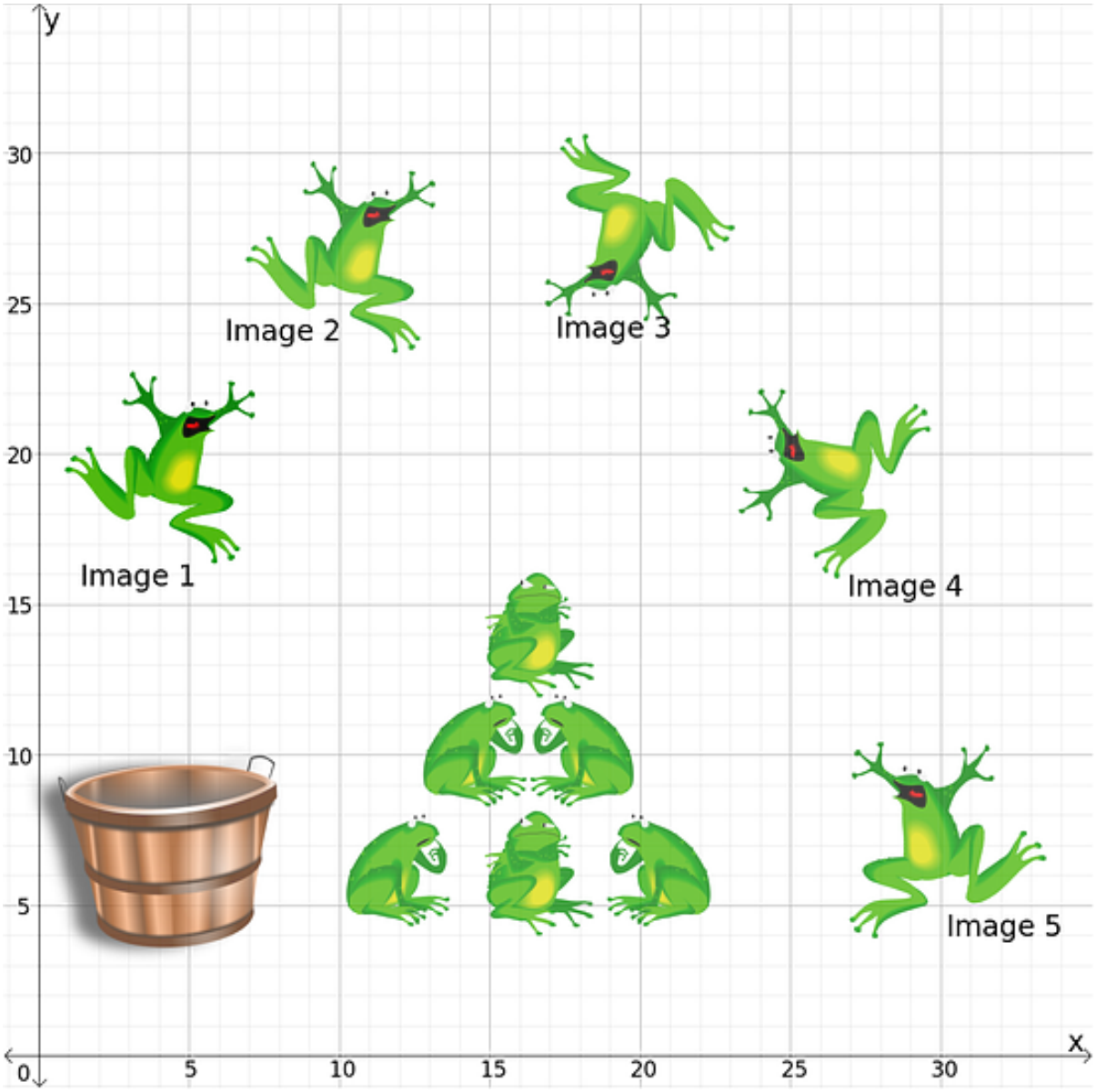
How do I justify that my proposed rigid transformation, or sequence of transformations, works?

Open Up the Math: Launch, Explore, Discuss

Josh is animating a scene in which a troupe of frogs is auditioning for the Animal Channel reality show, "The Bayou's Got Talent." In this scene the frogs are demonstrating their "leap frog" acrobatics act. Josh has completed a few key images in this segment, and now needs to describe the transformations that connect various images in the scene.

For each pre-image/image combination listed below, describe the transformation that moves the pre-image to the final image.

- If you decide the transformation is a rotation, you will need to give the center of rotation, the direction of the rotation (clockwise or counterclockwise), and the measure of the angle of rotation.
- If you decide the transformation is a reflection, you will need to give the equation of the line of reflection.
- If you decide the transformation is a translation, you will need to describe the "rise" and "run" between pre-image points and their corresponding image points.
- If you decide it takes a combination of transformations to get from the pre-image to the final image, describe each transformation in the order they would be completed.

1. 

Pre-image	Final Image	Description
image 1	image 2	
image 2	image 3	
image 3	image 4	
image 1	image 5	
image 2	image 4	

Ready for More?

How would you respond to these questions: Is it possible to find a sequence of transformations that will carry every image to every other image in the diagram if the first transformation in the sequence is always to translate the tip of the middle finger of the left hand of the first image to the corresponding point on the second image? Why or why not?

Takeaways

While working today on the rigid transformations _____

I noticed _____

I also noticed _____

and I am still wondering _____

Vocabulary

- coincides (superimposed or carried onto)
- corresponding points / sides
- **orientation**

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we identified the transformation, or sequence of transformations, that would carry one image onto another. We justified our claims by describing or showing the essential features of each transformation, such as the center of rotation or the line of reflection.

Unit 6, Lesson 4

Leap Year

Jump Start

Today's task contains this calendar page for February 28. For each definition, draw and label a diagram to show the meaning of the words.

February 28
A circle is the set of all points in a plane that are equidistant from a fixed point called the center of the circle.
An angle is the union of two rays that share a common endpoint.
An angle of rotation is formed when a ray is rotated about its endpoint. The ray that marks the pre-image of the rotation is referred to as the "initial ray" and the ray that marks the image of rotation is referred to as the "terminal ray."
Angle of rotation can also refer to the number of degrees a figure has been rotated around a fixed point, with a counterclockwise rotation being considered a positive direction of rotation.



1. A circle...
2. An angle...
3. An angle of rotation...
4. The calendar page for March 1 discusses some interesting facts about degree measurement, but doesn't define it. What is a degree?

Learning Focus

Write precise definitions of the rigid transformations.

How do I use my intuition, and the insights gained during the past few tasks, to identify or produce a rigid transformation?

How can I make my intuitive and insightful thinking explicit in words and diagrams?

What can I add to the words **slide**, **flip**, and **turn** to more precisely define the rigid transformations—translation, reflection, and rotation?

Open Up the Math: Launch, Explore, Discuss

What is wrong with **slide**, **flip**, and **turn** as words for defining the rigid transformations?

Slide, **flip,** and **turn** are verbs; they describe an action.

Here are ways these words might be used in a sentence:

Slide: "Slide over here for a minute."

Flip: "Flip the card over, so we can see which card it is."

Turn: "Turn the lid counterclockwise to open the jar."

1. Think about the work you did in class with *Leaping Lizards* and *Leap Frog*. What is missing in each of these sentences that would need to be included to make these actions a **translation**, a **reflection,** or a **rotation**?

The word "flip" can create a misconception—a wrong way of thinking.

2. Draw a picture to illustrate the sentence, "Flip the card over, so we can see which card it is." (Be as accurate as possible when drawing the two pictures of the card.)

3. Does your picture show a reflection? Why or why not?

Carlos and Clarita are discussing their latest business venture with their friend Juanita. They have created a daily planner that is both educational and entertaining. The planner consists of a pad of 365 pages bound together, one page for each day of the year. The planner is entertaining because images along the bottom of the pages form a flip-book animation when thumbed through rapidly. The planner is educational because each page contains some interesting facts. Each month has a different theme, and the facts for the month have been written to fit the theme. For example, the theme for January is astronomy, the theme for February is mathematics, and the theme for March is ancient civilizations. Carlos and Clarita have learned a lot from researching the facts they have included, and they have enjoyed creating the flip-book animation.

The twins are excited to share the prototype of their planner with Juanita before sending it to printing. Juanita, however, has a major concern. "Next year is leap year," Juanita explains. "You need 366 pages."

So now Carlos and Clarita have the dilemma of needing to create an extra page to insert between February 28 and March 1.

Here are the planner pages they have already designed.

February 28	**March 1**
A circle is the set of all points in a plane that are equidistant from a fixed point called the center of the circle. An angle is the union of two rays that share a common endpoint. An angle of rotation is formed when a ray is rotated about its endpoint. The ray that marks the pre-image of the rotation is referred to as the "initial ray" and the ray that marks the image of rotation is referred to as the "terminal ray." Angle of rotation can also refer to the number of degrees a figure has been rotated around a fixed point, with a counterclockwise rotation being considered a positive direction of rotation. 	Why are there 360° in a circle? One theory is that ancient astronomers established that a year was approximately 360 days, so the sun would advance its path, relative to the earth, approximately $1/360$ of a turn, or one degree, each day. (The 5 extra days in a year were considered unlucky days.) Another theory is that Babylonians first divided a circle into parts by inscribing a hexagon consisting of six equilateral triangles inside a circle. The angles of the equilateral triangles, located at the center of the circle, were further divided into 60 equal parts, since the Babylonian number system was base-60 (instead of base-10, like our number system). Another reason for 360° in a circle may be the fact that 360 has 24 divisors, so a circle can easily be divided into many smaller, equal-sized parts. 

Since February's theme is mathematics, Clarita suggests that they write formal definitions of the three rigid-motion transformations they have been using to create the images for the flip-book animation.

How would you complete each of the following definitions?

4. A **translation** of a set of points in a plane…

5. A **rotation** of a set of points in a plane…

6. A **reflection** of a set of points in a plane…

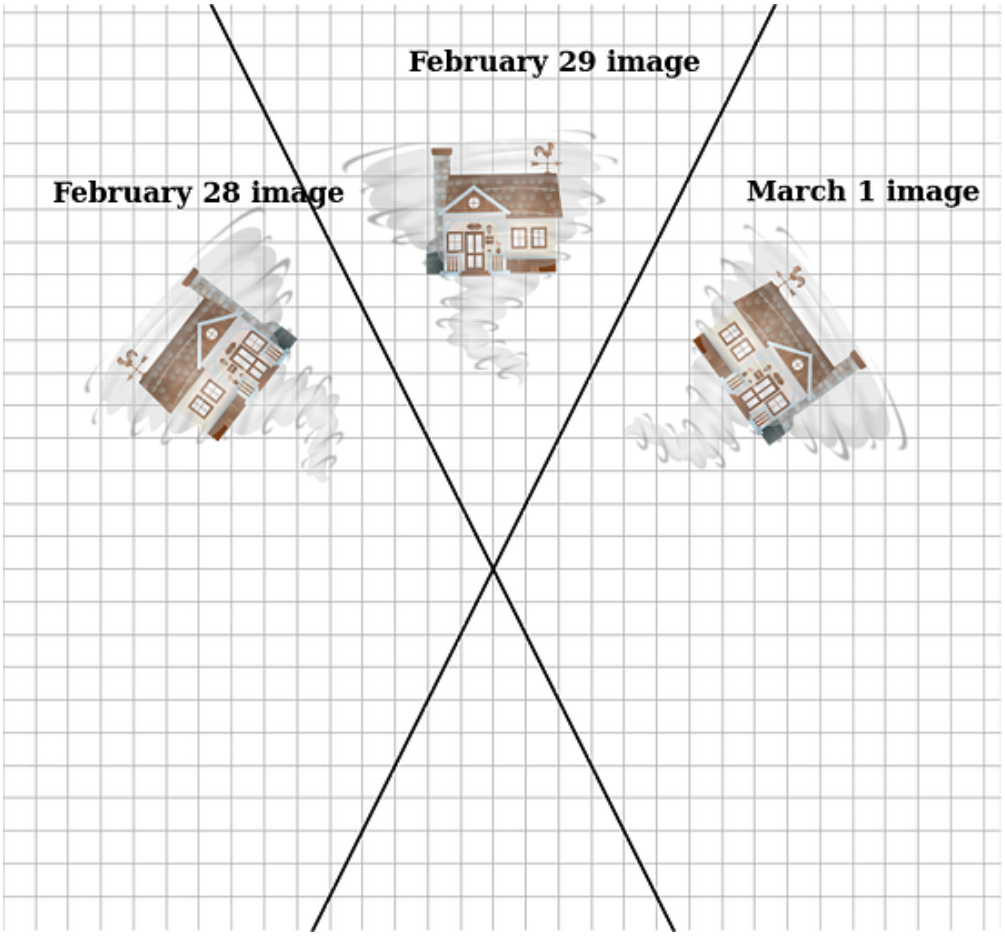
7. Complete:

   Translations, rotations, and reflections are **rigid transformations** because...

Note: Carlos and Clarita used these words and phrases in their definitions: **perpendicular bisector**, center of rotation, equidistant, angle of rotation, concentric circles, parallel, image, pre-image, **preserves distance and angle measures** within the shape. Revise your definitions to include these words or phrases.

Ready for More?

In addition to writing new facts for February 29, the twins also need to add another image in the middle of their flip-book animation. The animation sequence is of Dorothy's house from *The Wizard of Oz* as it is being carried over the rainbow by a tornado. The house in the February 28 drawing has been rotated to create the house in the March 1 drawing. Carlos believes that they can get from the February 28 drawing to the March 1 drawing by reflecting the February 28 drawing, and then reflecting it again.



Verify that the image Carlos inserted between the two images that appeared on February 28 and March 1 works as they intended. For example:

- What convinces you that the February 29 image is a reflection of the February 28 image about the given line of reflection?
- What convinces you that the March 1 image is a reflection of the February 29 image about the given line of reflection?
- What convinces you that the two reflections together complete a rotation between the February 28 and March 1 images?

Takeaways

The essential elements of each definition are as follows:

- Translation: _____
- Rotation: _____
- Reflection: _____

Vocabulary

- **angle**
- **angle of rotation**
- **circle**
- **concentric circles**
- **degree**
- **perpendicular bisector**
- **preserves distance and angle measure**
- **quadrilaterals: types**

Bold terms are new in this lesson.

Lesson Summary

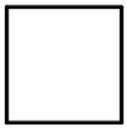
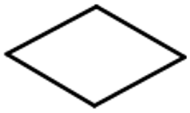
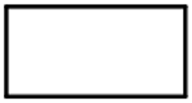
In this lesson, we wrote precise definitions of the three rigid-motion transformations: translation, rotation, and reflection, and explored why the words slide, turn, and flip were not adequate to use as definitions.

Unit 6, Lesson 5

Symmetries of Quadrilaterals

Jump Start

Which One Doesn't Belong?

A. A square 	B. A rhombus 
C. A rectangle 	D. An equilateral triangle 

Learning Focus

Identify transformations that carry an image onto itself.

What does it mean to say that a figure is symmetrical?

How is symmetry related to rigid transformations?

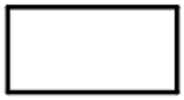
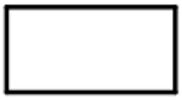
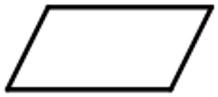
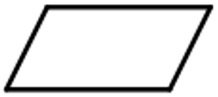
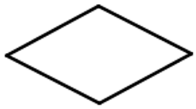
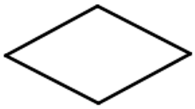
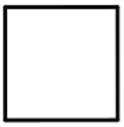
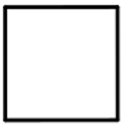
Open Up the Math: Launch, Explore, Discuss

A line that reflects a figure onto itself is called a line of symmetry. A figure that can be carried onto itself by a rotation has rotational symmetry.

Every four-sided polygon is a quadrilateral. Some quadrilaterals have additional properties and are given special names like squares, parallelograms, and rhombuses. A diagonal of a quadrilateral is formed when opposite vertices are connected by a line segment. Some quadrilaterals are symmetric about their diagonals. Some are symmetric about other lines. In this task you will use rigid-motion transformations to explore line symmetry and rotational symmetry in various types of quadrilaterals.

For each of the following quadrilaterals you are going to try to answer the question, "Is it possible to reflect or rotate this quadrilateral onto itself?" As you experiment with each quadrilateral, record your findings in the following chart. Be as specific as possible with your descriptions.

1. Complete:

Defining features of the quadrilateral	Lines of symmetry that reflect the quadrilateral onto itself	Center and angles of rotation that carry the quadrilateral onto itself
A rectangle is a quadrilateral that contains four right angles.		
A parallelogram is a quadrilateral in which opposite sides are parallel.		
A rhombus is a quadrilateral in which all sides are congruent.		
A square is both a rectangle and a rhombus.		

Pause and Reflect

Consider the set of quadrilaterals that are trapezoids, with exactly one pair of opposite sides parallel. Is it possible to reflect or rotate such a trapezoid onto itself?

Draw a trapezoid with exactly one pair of parallel sides. Then see if you can find:

- any lines of symmetry, or
- any centers of rotational symmetry,

that will carry the trapezoid you drew onto itself.

If you were unable to find a line of symmetry or a center of rotational symmetry for your trapezoid, see if you can sketch a different trapezoid with exactly one pair of parallel sides that might possess some type of symmetry.

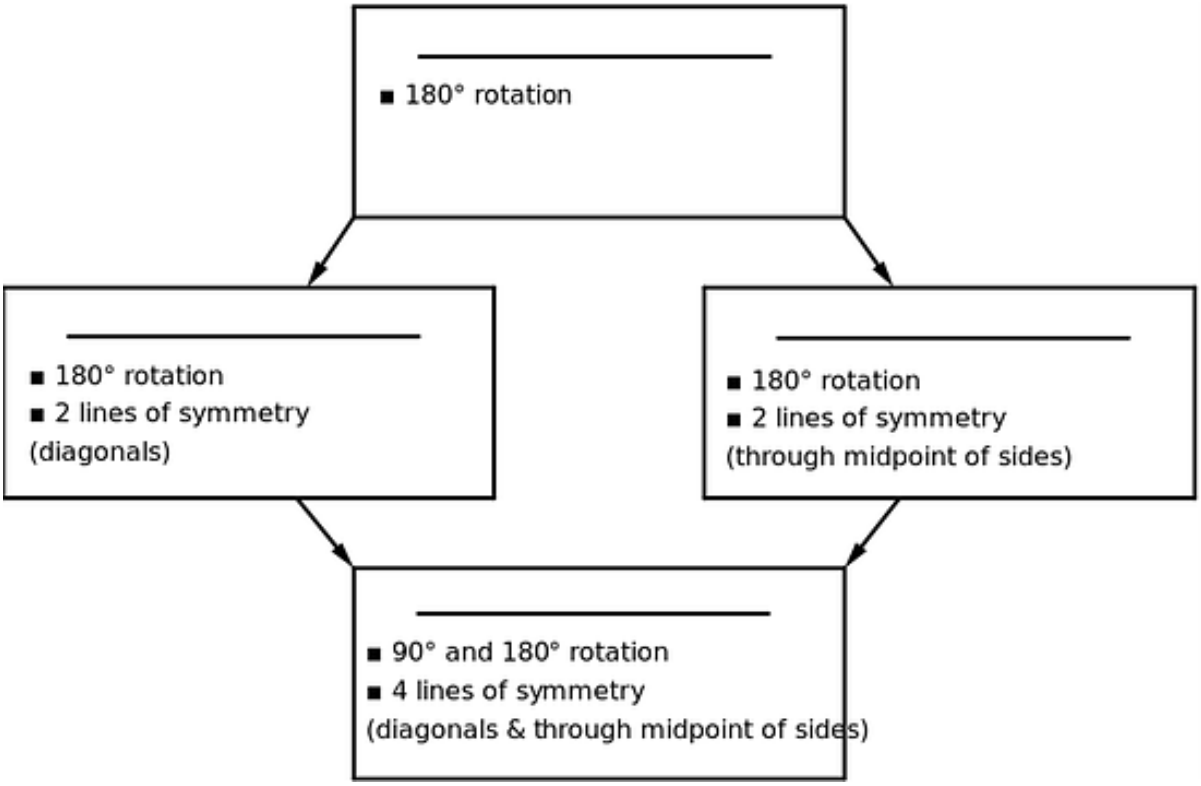
Ready for More?

Can you find other polygons that have rotational or line symmetry? For example:

1. Can you draw a quadrilateral that has only one line of symmetry?
2. Can you draw a polygon that has three lines of symmetry?
3. Can you draw a polygon that has rotational symmetry of $40°$?

Takeaways

We have found that many different quadrilaterals possess lines of symmetry and/or rotational symmetry. In the following chart, write the names of the quadrilaterals that are being described in terms of their symmetries.



What do you notice about the relationships between quadrilaterals based on their symmetries and highlighted in the structure of the above chart?

Adding Notation, Vocabulary, and Conventions

We use the notation _____ to represent the line segment with endpoints at point A and point B. If we want to refer to the length of the line segment, or the distance between the endpoints, we use the symbol _____. Therefore, _____ is a geometric object (a line segment), while _____ is a number (a distance or a length).

If two line segments, _____ and _____, are the same length, they can be moved using translations, reflections, and rotations until their endpoints match. We say that the two line segments are congruent, and we write this symbolically as _____. However, we use the equal sign to show that the line segments are the same length, _____, since lengths are numbers rather than geometric objects.

Angles are represented by _____, using a single letter, the vertex of the angle; or by _____, using three letters, including the vertex, if there might be more than one angle with the same vertex. The measure of the angle is referred to by the symbol _____.

Vocabulary

- **congruent (CPCTC)**
- **diagonal**
- equilateral, equilateral triangle

- hexagon
- **line of symmetry**
- midpoint
- octagon
- opposite angles, opposite vertices
- opposite sides (in a parallelogram or an even-sided polygon)
- parallelogram
- pentagon
- polygon
- **quadrilaterals: types**
- rectangle
- **regular polygon**
- rhombus
- **rotational symmetry**
- square
- symmetry
- **trapezoid**

Bold terms are new in this lesson.

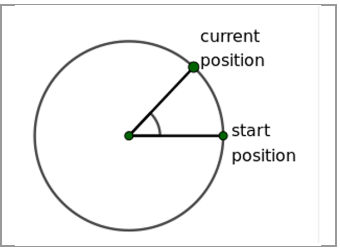
Lesson Summary

In this lesson, we explored line and rotational symmetry in different types of quadrilaterals. A figure is symmetric if a figure can be reflected across a line or rotated about a point onto itself. We found that diagonals and lines connecting the midpoints of opposite sides of a quadrilateral might be lines of symmetry, depending on the quadrilateral, and the point of intersection of the diagonals is the center of rotation for parallelograms, rectangles, rhombuses, and squares. The possible angles of rotation vary depending on the quadrilateral, but are always multiples of 90°.

Unit 6, Lesson 6

Symmetries of Regular Polygons

Jump Start

You are running around a circular track. What fraction of a lap around the track have you run if the angle formed by your starting position, the center of the track, and your current position measures:	

1. 180°
2. 45°
3. 270°
4. 36°
5. 72°

Learning Focus

Find patterns of line and rotational symmetry in regular polygons.

What makes a polygon regular or symmetric?

How does the symmetry of regular polygons differ depending on the number of sides?

What patterns can I find in the number and characteristics of the lines of symmetry in a regular n-gon (a polygon with n sides)?

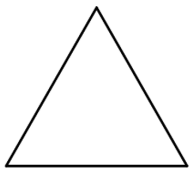
What patterns can I find that describe the nature of the rotational symmetry in a regular n-gon?

Open Up the Math: Launch, Explore, Discuss

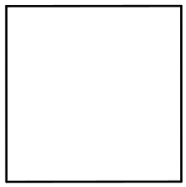
A line that reflects a figure onto itself is called a **line of symmetry**. A figure that can be carried onto itself by a rotation is said to have **rotational symmetry**. A **diagonal of a polygon** is any line segment that connects non-consecutive vertices of the polygon.

For each of the following regular polygons, describe the rotations and reflections that carry it onto itself (be as specific as possible in your descriptions, such as specifying the angle of rotation).

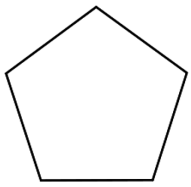
1. An equilateral triangle



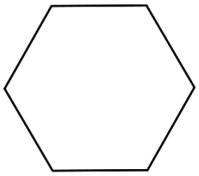
2. A square



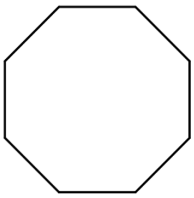
3. A regular pentagon



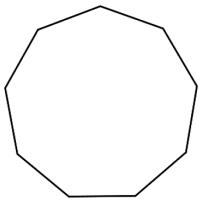
4. A regular hexagon



5. A regular octagon



6. A regular nonagon



7. What patterns do you notice in terms of the number and characteristics of the lines of symmetry in a regular polygon?

8. What patterns do you notice in terms of the angles of rotation when describing the rotational symmetry in a regular polygon?

Ready for More?

You may have found a rule for the number of lines of symmetry numerically—by looking for patterns in an input-output table relating the number of sides of the polygon to the number of lines of symmetry. Can you justify why this conjecture is correct, since you haven't tried all possible regular polygons?

Write a detailed argument to a friend explaining why there are n lines of symmetry in a regular polygon, even though regular polygons with an odd number of sides have different types of lines of symmetry from those found in regular polygons with an even number of sides. Use precise and accurate language, and support your words with appropriate diagrams and illustrations.

Trade your paper with another student and read and critique each other's explanations.

Takeaways

I conjectured that the number of lines of symmetry in a regular polygon with n sides is _____

I can explain this by:

I conjectured that the smallest angle of rotation in a regular polygon with n sides can be found by _____

This is because:

Vocabulary

- **bisect (verb); bisector (noun) (midpoint)**
- **midpoint**
- **n-gon**
- **opposite angles, opposite vertices**
- **opposite sides (in a parallelogram or an even-sided polygon)**

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we examined lines of symmetry and rotational symmetry in regular polygons. We found that the number of lines of symmetry and the smallest angle of rotation could be related to the number of sides of the regular polygon.

Unit 6, Lesson 7
Quadrilaterals: Beyond Definition

Jump Start

Give the name of a geometric figure that has the following characteristics:

1. A type of quadrilateral with no lines of symmetry.

2. A type of quadrilateral whose diagonals are its only lines of symmetry.

3. A type of quadrilateral where the lines through the midpoints of opposite sides are lines of symmetry.

Learning Focus

Relate attributes of special quadrilaterals to symmetry.

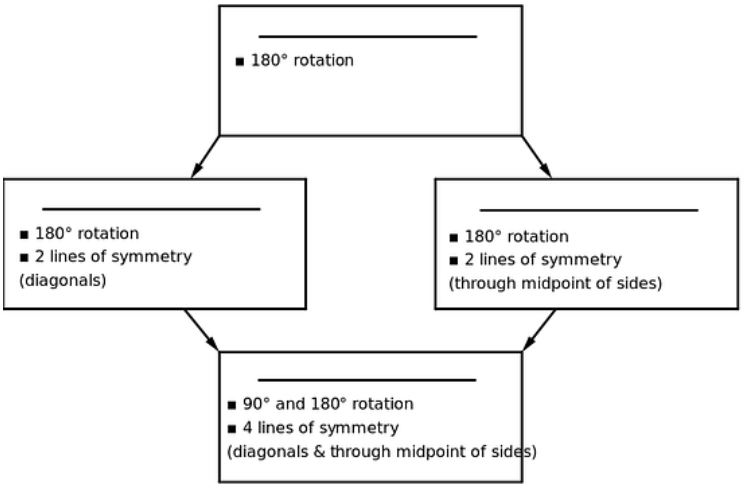
What else might be true about parallelograms, rectangles, squares, or rhombuses other than the characteristics given about them in their definitions?

How might I be convinced that certain characteristics must occur in every member of a special class of quadrilaterals?

Open Up the Math: Launch, Explore, Discuss

1. We have found that many different quadrilaterals possess lines of symmetry and/or rotational symmetry. In the following chart, write the names of the quadrilaterals that are being described in terms of their symmetries.

   Based on the symmetries we have observed in various types of quadrilaterals, we can make claims about other features and properties that the quadrilaterals may possess.



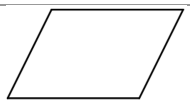
2. A **rectangle** is a quadrilateral that contains four right angles.

   Based on what you know about transformations, what else can we say about rectangles besides the defining property that "all four angles are right angles?" Make a list of additional properties of rectangles that seem to be true (we call such statements conjectures) based on the transformation(s) of the rectangle onto itself. You will want to consider properties of the sides, the angles, and the diagonals. Then justify why the properties would be true for this specific example of a rectangle using transformational symmetry.



3. A **parallelogram** is a quadrilateral in which opposite sides are parallel.

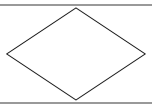
   Based on what you know about transformations, what else can we say about parallelograms besides the defining property that "opposite sides of a parallelogram are parallel?" Make a list of additional properties of parallelograms that seem to be true based on the transformation(s) of the parallelogram onto itself. You will want to consider properties of the sides, angles, and diagonals. Then justify why the properties would be true for this specific example of a parallelogram using transformational symmetry.



Pause and Reflect

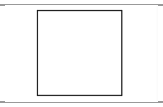
4. A **rhombus** is a quadrilateral in which all four sides are congruent.

   Based on what you know about transformations, what else can we say about a rhombus besides the defining property that "all sides are congruent?" Make a list of additional properties of rhombuses that seem to be true based on the transformation(s) of the rhombus onto itself. You will want to consider properties of the sides, angles, and diagonals. Then justify why the properties would be true for this specific example of a rhombus using transformational symmetry.



5. A **square** is both a rectangle and a rhombus.

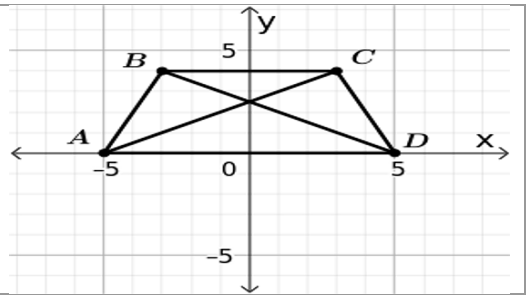
   Based on what you know about transformations, what can we say about a square? Make a list of properties of squares that seem to be true based on the transformation(s) of the squares onto itself. You will want to consider properties of the sides, angles, and diagonals. Then justify why the properties would be true for this specific example of a square using transformational symmetry.



Ready for More?

An **isosceles trapezoid** is a quadrilateral with one pair of parallel sides and the non-parallel sides are congruent, as shown in figure $ABCD$. Can you find a way to demonstrate that the following statements are true?

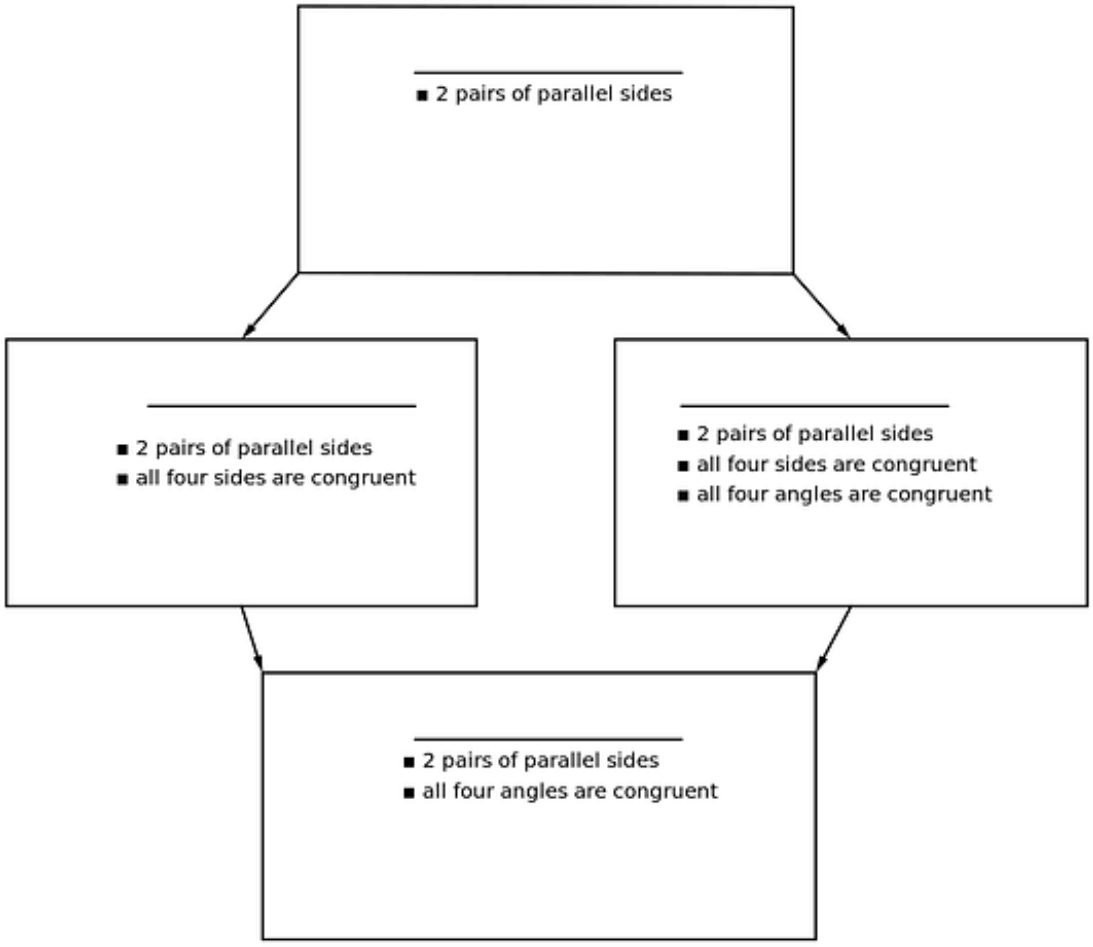
- The diagonals do not bisect each other.
- The diagonals are congruent.
- $\angle BAD$ and $\angle CBA$ are supplementary, that is, $m\angle BAD + m\angle CBA = 180°$.



Takeaways

Our Conjectures about Properties of Quadrilaterals (based on experimentation and reasoning with rigid transformations)

In the following chart, write the names of the quadrilaterals that are being described in terms of their features and properties, and then record any additional features or properties of that type of quadrilateral you may have observed. Be prepared to share reasons for your observations.



What do you notice about the relationships between quadrilaterals based on their characteristics and the structure of the above chart?

How are the charts at the beginning and end of this task related? What do they suggest?

Vocabulary

- **conjecture**
- quadrilaterals: types

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we used rigid transformations to examine properties of the sides, angles, and diagonals in parallelograms, rectangles, rhombuses, and squares. We learned that some quadrilaterals can be classified in terms of the properties they share with other quadrilaterals, such as congruent opposite sides or angles.