Unit 5, Lesson 1

Business Cents

Learning Focus

Solve systems of linear equations that represent a context.

What strategies might I use to find the point of intersection of two linear equations?

What does each strategy reveal? What might each strategy conceal?

How do I use the solution to a system of equations to make decisions within a context?

Open Up the Math: Launch, Explore, Discuss

The Martinez twins, Carlos and Clarita, are trying to find a way to make money during summer vacation. When they overhear their aunt complaining about how difficult it is to find someone to care for her pets while she will be away on a trip, Carlos and Clarita know they have found the perfect solution. Not only do they have a large, unused storage shed on their property where they can house animals, they also have a spacious fenced backyard where the pets can play.

Carlos and Clarita need to clean the storage shed where they plan to board the pets. They have decided to hire a company to clean the windows. After collecting the following information, they have come to you for help deciding which window cleaning company they should hire.

- *Sunshine Express Window Cleaners* charges $\$50$ for each service call, plus $\$10$ per window.
- *"Pane"less Window Cleaners* charges $\$25$ for each service call, plus $\$15$ per window.

1. Which company would you recommend, and why? Prepare an argument to convince Carlos and Clarita that your recommendation is reasonable. (It is always more convincing if you can support your claim in multiple ways. How might you support your recommendation using a table? A graph? Algebra?)

Once the windows are clean, Carlos and Clarita also want to have the window frames repainted to make the storage shed look more appealing to potential customers. They have decided to hire a company to paint the windows. Again, they need your help deciding which painter they should hire.

Paint by Numbers charges $\$100$ for sanding the frames and masking the windows with tape, and $\$16$ per hour for painting, since different employees do different parts of the work.

Paint it Right charges $\$24$ per hour for all labor, plus a $\$38$ waste disposal fee.

2. Which company would you recommend, and why? Prepare an argument to convince Carlos and Clarita that your recommendation is reasonable. (Support your claim in multiple ways including a table, a graph, and algebra.)

Both paint companies allow the customer to purchase their own paint supplies. Carlos has done some research and found the following information:

Handyman Tools and Supplies sells a small painter's kit that contains 2 paint brushes and 4 rolls of masking tape for $\$16$. They also sell a large painter's kit that contains 5 paint brushes and 15 rolls of masking tape for $\$50$.

Carlos's dad is planning to paint a couple of rooms in their house, and has offered to buy any left-over supplies. Carlos is trying to figure out how much he would charge his dad for each paint brush and each roll of tape. To keep track of the information, he has written the following equations:

$$2p + 4r = 16$$
$$5p + 15r = 50$$

Clarita is looking at Carlos's notes and asks, "What do p and r stand for?"

3. How would you answer Clarita's question?

Clarita says, "I think I can find the values of p and r using a table or a graph."

Carlos says, "I think I can find the values of p and r using algebra."

4. Solve for p and r using both a graphical strategy and an algebraic strategy.

Ready for More?

While solving the system of equations about the painting supplies, what assumptions did you make about the context? Are these assumptions necessarily true?

Takeaways

While each equation in a system of equations has many solutions, the solution to a system of linear equations is:

Systems of equations can be solved using tables, graphs or algebra. The solution to the system is revealed in different ways, and each strategy has its own pros and cons.

Using a graph: How the solution is revealed: Pros: Cons:	**Using a table:** How the solution is revealed: Pros: Cons:

Using algebra:

How the solution is revealed:

Pros:

Cons:

There are two methods for solving a system of linear equations algebraically by substitution:

Method 1:

Method 2:

These methods work equally well for systems in which the linear equations are linear functions, and for systems in which the linear equations are linear combinations with a constant sum.

A linear combination is: _____

Vocabulary

- **linear combination**
- linear function

- **satisfies an equation**
- system of equations

Lesson Summary

In this lesson, we reviewed strategies for solving systems of linear equations, including using tables, graphs, and algebra. We examined two different algebraic methods, both of which involve substitution of expressions for variables. These methods work with systems of linear equations that are linear functions, as well as systems of linear equations that are linear combinations.

Unit 5, Lesson 2

Pet Sitters

Learning Focus

Represent linear constraints.

How can I represent all possible solutions to a situation that is limited in different ways by various resources or constraints?

Open Up the Math: Launch, Explore, Discuss

Carlos and Clarita are making a list of some of the issues they need to consider as part of their business plan to care for cats and dogs while their owners are on vacation.

- **Space**: Cat pens will require 6 ft^2 of space, while dog runs require 24 ft^2. Carlos and Clarita have up to 360 ft^2 available in the storage shed for pens and runs, while still leaving enough room to move around the cages.
- **Start-up costs:** Carlos and Clarita plan to invest much of the $\$1,280$ they earned from their last business venture to purchase cat pens and dog runs. It will cost $\$32$ for each cat pen and $\$80$ for each dog run.

Of course, Carlos and Clarita want to make as much money as possible from their business, so they are trying to determine how many of each type of pet they should plan to accommodate. They plan to charge $\$8$ per day for boarding each cat and $\$20$ per day for each dog.

After surveying the community regarding the pet boarding needs, Carlos and Clarita are confident that they can keep all of their boarding spaces filled for the summer.

So, the question is, how many of each type of pet should they prepare for? Their dad has suggested the same number of each, perhaps 12 cats and 12 dogs. Carlos thinks they should plan for more dogs, since they can charge more. Clarita thinks they should plan for more cats since they take less space and time, and therefore, they can board more.

1. What do you think? What recommendations would you give to Carlos and Clarita, and what argument would you use to convince them that your recommendation is reasonable?

Ready for More?

Try a different, and perhaps more challenging, representation than you are currently using by considering the following questions:

How can both constraints be considered simultaneously?

How might you represent all the possible combinations of cats and dogs that Carlos and Clarita can take care of on a graph? Symbolically?

Takeaways

In *Pet Sitters*, the statements about start-up costs and space are called constraints because _____.

Viable options for the space constraint include _____, because _____.

The *Pet Sitters* context can be modeled with a system of linear inequalities, because _____.

In the *Pet Sitters* context, Carlos and Clarita want to maximize their profit. The statement about profit is not a constraint because,_____.

Instead, profit is the result of _____.

Vocabulary

- **constraint**
- **system of inequalities**
- **viable, non-viable options**

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we encountered a situation in which decisions were limited by certain constraints. We tested combinations of quantities to see if they fit one, both, or neither constraint. Although we didn't resolve the situation today, we recognized the goal of this type of work: to make decisions within the parameters of the constraints that will maximize (or minimize) a quantity, such as profit.

Unit 5, Lesson 3

Too Big or Not Too Big, That Is the Question

Jump Start

1. Find at least 5 coordinate pairs (x, y) that make the following inequality true:

   $2x + 4y < 20$

Represent each of the following statements algebraically using inequality symbols and variables to represent the quantities described in the problems: (Remember to define the meaning of the variables you use in your inequalities.)

2. It will cost at least $\$50$ to purchase two different video games for the evening activity.

3. We expect that at most 60 children and adults will attend the family reunion.

4. No more than 30 minutes should be spent preparing breakfast and lunch.

5. The total cost for a morning and afternoon snack should not exceed $\$10$.

Learning Focus

Graph the solution set for linear inequalities in two variables.

How can I find the complete set of points that satisfy a given constraint?

How do I represent the complete solution set?

Open Up the Math: Launch, Explore, Discuss

As Carlos is considering the amount of money available for purchasing cat pens and dog runs he realizes that his father's suggestion of boarding "the same number of each, perhaps 12 cats and 12 dogs" may not be reasonable. Why?

- Start-up costs: Carlos and Clarita plan to invest much of the $\$1,280$ they earned from their last business venture to purchase cat pens and dog runs. It will cost $\$32$ for each cat pen and $\$80$ for each dog run.

1.

   a. Find at least 5 more combinations of cats and dogs that would be "too big" based on this start-up cost constraint. Plot each of these combinations as points on a coordinate grid using the same color for each point.

   b. Find at least 5 combinations of cats and dogs that would be "not too big" based on this start-up cost constraint. Plot each of these combinations as points on a coordinate grid using a different color for the points than you used in 1a.

   c. Find at least 5 combinations of cats and dogs that would be "just right" based on this start-up cost constraint. That is, find combinations of cat pens and dog runs that would cost exactly $\$1,280$. Plot each of these combinations as points on a coordinate grid using a third color.

2. What do you notice about these three different collections of points?

3. Write an equation for the line that passes through the points representing combinations of cat pens and dog runs that cost exactly $1,280. What does the slope of this line represent?

Carlos and Clarita don't have to spend all of their money on cat pens and dog runs, unless it will help them maximize their profit.

4. Shade all of the points on your coordinate grid that **satisfy** the start-up costs constraint.

5. Write a mathematical rule to represent the points shaded in problem 4. That is, write an inequality whose **solution set** is the collection of points that satisfy the start-up costs constraint.

In addition to start-up costs, Carlos needs to consider how much space he has available, based on the following:

- **Space**: Cat pens will require 6 ft^2 of space, while dog runs require 24 ft^2. Carlos and Clarita have up to 360 ft^2 available in the storage shed for pens and runs, while still leaving enough room to move around the cages.

6. Write an inequality to represent the solution set for the space constraint. Shade the solution set for this inequality on a different coordinate grid.

What do you think? What recommendation would you give to Carlos and Clarita regarding how many cats and dogs to plan on boarding, and what argument would you use to convince them that your recommendation is reasonable?

Ready for More?

You can come up with the equation of a boundary line using many different strategies. Try to write the equation of the boundary line for the space constraint in each of the following ways: (You may have to find some additional information to try each strategy.)

1. Directly from the words used to describe the constraint.
2. Using just the x- and y-intercepts.
3. Using any two points you might have found on the boundary line.
4. Using the "exchange rate" between cats and dogs.

Takeaways

Linear equations can be written in the following forms:

Standard form: _____

Slope-intercept form: _____

Point-slope form: _____

The solution set to a linear inequality is a half plane, that is, _____.

The solution set to an inequality constraint for a context like *Pet Sitters* is _____.

The boundary line for a linear constraint may be written in _____ form to reveal _____, or _____ form to reveal _____.

Vocabulary

- **half-plane**
- **linear function**

Lesson Summary

In this lesson, we learned how to find and represent all of the points in the solution set for a linear inequality in two variables.

Unit 5, Lesson 4

Some of One, None of the Other

Learning Focus

Graph linear equations in standard form.

Why is it useful to use equivalent forms of linear equations, and how do I convert a linear equation from one form to the other?

Open Up the Math: Launch, Explore, Discuss

Carlos and Clarita are comparing strategies for writing equations of the boundary lines for the *Pet Sitters* constraints. They are discussing their work on the **space** constraint.

- **Space**: Cat pens will require 6 ft^2 of space, while dog runs require 24 ft^2. Carlos and Clarita have up to 360 ft^2 available in the storage shed for pens and runs, while still leaving enough room to move around the cages.

Carlos's Method: "I made a table. If I don't have any cats, then I have room for 15 dogs. If I use some of the space for 4 cats, then I can have 14 dogs. With 8 cats, I have room for 13 dogs. For each additional dog run that I don't buy, I can buy 4 more cat pens. From my table I know the y-intercept of my line is 15 and the slope is $-\frac{1}{4}$, so my equation is $y = -\frac{1}{4}x + 15$."

Clarita's Method: "I let x represent the number of cats, and y the number of dogs. Since cat pens require 6 ft^2, $6x$ represents the space used by cats. Since dog runs require 24 ft^2, $24y$ represents the amount of space used by dogs. So, my equation is $6x + 24y = 360$."

1. Since both equations represent the same information, they must be equivalent to each other.

    a. Show the steps you could use to turn Clarita's equation into Carlos's equation. Explain why you can do each step.

    b. Show the steps you could use to turn Carlos's equation into Clarita's. Explain why you can do each step.

2. Use both Carlos's and Clarita's methods to write the equation of the boundary line for the start-up costs constraint.

    a. Carlos's method:

    b. Clarita's method:

Start-up costs: Carlos and Clarita plan to invest much of the $\$1,280$ they earned from their last business venture to purchase cat pens and dog runs. It will cost $\$32$ for each cat pen and $\$80$ for each dog run.

3. Show the steps you could use to turn Clarita's start-up costs equation into Carlos's equation. Explain why you can do each step.

4. Show the steps you could use to turn Carlos's start-up costs equation into Clarita's. Explain why you can do each step.

Pause and Reflect

In addition to writing an equation of the boundary lines, Carlos and Clarita need to graph their lines on a coordinate grid.

Carlos's equation is written in slope-intercept form. Clarita's equation is written in standard form. Both forms are ways of writing linear equations.

Both Carlos and Clarita know they only need to plot two points in order to graph a line.

5. Carlos's strategy: How might Carlos use his slope-intercept form, $y = -\frac{1}{4}x + 15$, to plot two points on his line?

6. Clarita's strategy: How might Clarita use her standard form, $6x + 24y = 360$, to plot two points on her line? (Clarita is really clever, so she looks for the two easiest points she can find.)

7. Write equations for the following two constraints:

   Space:

   Start-up costs:

   Find where the two lines intersect algebraically. Record enough steps so that someone else can follow your strategy.

8. What does this point mean in the context of cats and dogs?

Ready for More?

If you only know the x- and y-intercepts of a linear function, how can you find both the standard form and the slope-intercept form of the equation of the line?

Takeaways

Since two points determine a line, one strategy for graphing a linear equation is:

This strategy works particularly well for linear equations written in _____.

Another strategy for graphing a linear equation is:

This strategy works particularly well for linear equations written in _____.

Contexts that are best represented by linear equations in standard form contain information about: _____.

Contexts that are best represented by linear equations in slope-intercept form contain information about: _____.

Adding Notation, Vocabulary, and Conventions

The standard form for a linear equation is, by convention

To change slope-intercept form to standard form:

To change standard form to slope-intercept form:

Vocabulary

- linear function
- slope-intercept form of a line
- standard form of line

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we learned the conventions for writing the standard form of a linear equation and strategies for turning slope-intercept form into standard form and standard form into slope-intercept form. We also learned a new method for graphing linear equations in standard form by finding the intercepts.

Unit 5, Lesson 5

Pampering and Feeding Time

Learning Focus

Represent constraints symbolically and graphically.

What are efficient ways to write the inequalities and sketch the solution sets representing these additional constraints on feeding time and pampering time?

Open Up the Math: Launch, Explore, Discuss

Carlos and Clarita have been worried about space and start-up costs for their pet sitting business, but they realize they also have a limit on the amount of time they have for taking care of the animals they board. To keep things fair, they have agreed on the following time constraints.

- **Feeding time:** Carlos and Clarita estimate that cats will require 6 minutes twice a day—morning and evening—to feed and clean their litter boxes, for a total of 12 minutes per day for each cat. Dogs will require 10 minutes twice a day to feed and walk, for a total of 20 minutes per day for each dog. Carlos can spend up to 8 hours each day for the morning and evening feedings, but needs the middle of the day off for baseball practice and games.
- **Pampering time:** The twins plan to spend 16 minutes each day brushing and petting each cat, and 20 minutes each day bathing or playing with each dog. Clarita needs time off in the morning for swim team and evening for her art class, but she can spend up to 8 hours during the middle of the day to pamper and play with the pets.

1. Write each of these additional time constraints symbolically.

Feeding time constraint:	Pampering time constraint:

2. Shade the solution set for each constraint on separate coordinate grids.

Feeding time:	Pampering time:

3. Now find the point of intersection for the two time constraints, pampering time and feeding time.

4. What does this point mean in the context of cats and dogs?

Ready for More?

One week Carlos twisted his ankle during baseball practice, and so it required more time for him to feed and walk the dogs. Here is the feeding time constraint for that week:

- **Feeding time:** Each cat will require 12 minutes per day to feed and clean their litter boxes. Each dog will require 30 minutes per day to feed and walk them. Carlos can spend up to 8 hours each day doing these activities.
- **Pampering time:** Clarita will spend 16 minutes each day brushing and petting each cat, and 20 minutes each day bathing or playing with each dog. Clarita can spend up to 8 hours during the middle of the day to pamper and play with the pets.

1. Find the point of intersection for these revised time constraints.

Feeding time constraint:	Pampering time constraint:

Point of intersection:

2. What does this point mean in the context of cats and dogs?

Takeaways

Ways to determine which half plane should be shaded after graphing the boundary line for a linear inequality include:

Using different units for measuring the quantities in a constraint leads to:

A statement about a limitation or restriction in a modeling context may be eliminated as a constraint if:

Lesson Summary

In this lesson, we drew upon everything we have learned so far about modeling contexts with linear inequalities as constraints. We learned that in a system of constraints, quantities need to be measured in the same units, and that individual constraints are equivalent and produce the same solution set even if the quantities are measured in different units.

Unit 5, Lesson 6

All for One, One for All

Jump Start

Here are all of the constraints Carlos and Clarita have identified for their pet sitting business.

- **Space**: Cat pens will require 6 ft^2 of space, while dog runs require 24 ft^2. Carlos and Clarita have up to 360 ft^2 available in the storage shed for pens and runs, while still leaving enough room to move around the cages.
- **Start-up costs:** Carlos and Clarita plan to invest much of the $\$1,280$ they earned from their last business venture to purchase cat pens and dog runs. It will cost $\$32$ for each cat pen and $\$80$ for each dog run.
- **Feeding time:** Carlos and Clarita estimate that cats will require 6 minutes twice a day—morning and evening—to feed and clean their litter boxes, for a total of 12 minutes per day for each cat. Dogs will require 10 minutes twice a day to feed and walk, for a total of 20 minutes per day for each dog. Carlos can spend up to 8 hours each day for the morning and evening feedings, but needs the middle of the day off for baseball practice and games.
- **Pampering time:** The twins plan to spend 16 minutes each day brushing and petting each cat, and 20 minutes each day bathing or playing with each dog. Clarita needs time off in the morning for swim team and evening for her art class, but she can spend up to 8 hours during the middle of the day to pamper and play with the pets.

Write the system of inequalities that model these constraints.

Learning Focus

Find the solution set for a system of linear inequalities.

How do I represent the solutions that satisfy all of the constraints being placed on a context?

Open Up the Math: Launch, Explore, Discuss

Carlos and Clarita have found a way to represent combinations of cats and dogs that satisfy each of their individual *Pet Sitters* constraints, but they realize that they need to find combinations that satisfy all of the constraints simultaneously. Why?

1. Begin by listing the **system of inequalities** you have written to represent the **start-up costs** and **space** *Pet Sitters* constraints.
2. Find at least 5 combinations of cats and dogs that would satisfy both of the constraints represented by this system of inequalities. How do you know these combinations work?
3. Find at least 5 combinations of cats and dogs that would satisfy one of the constraints, but not the other. For each combination, explain how you know it works for one of the inequalities, but not for the other?
4. Shade a region on the coordinate grid that would represent the solution set for the system of inequalities. Explain how you found the region to shade.

Pause and Reflect

5. Rewrite your systems of inequalities to include the additional constraints for *feeding time* and *pampering time*.
6. Find at least five combinations of cats and dogs that would satisfy all of the constraints represented by this new system of inequalities. How do you know these combinations work?

7. Find at least five combinations of cats and dogs that would satisfy some of the constraints, but not all of them. For each combination, explain how you know it works for some inequalities, but not for others?

8. Shade a region of the coordinate grid that would represent the solution set to the system of inequalities consisting of all four *Pet Sitters* constraints. Explain how you found the region to shade.

9. Shade a region in Quadrant I of the coordinate grid that would represent all possible combinations of cats and dogs that satisfy the four *Pet Sitters* constraints. This set of points is referred to as the feasible region since Carlos and Clarita can feasibly board any of the combinations of cats and dogs represented by the points in this region without exceeding any of their constraints on time, money, or space.

10. How is the feasible region shaded in problem 9 different from the solution set to the system of inequalities shaded in problem 8?

Ready for More?

Graph the solution set for the following system of inequalities. To clearly define the solution set, you will also want to find the vertices of the polygon that represents the feasible region for the solutions of the system.

$$\begin{cases} 2x + y \geq 4 \\ \frac{1}{2}x + y \leq 2 \\ x - y \leq 4 \end{cases}$$

Takeaways

To find the solution for a system of inequalities:

To define the feasible region for a system of constraints that models a context:

Adding Notation, Vocabulary, and Conventions

Given a system of inequalities $\begin{cases} A_1x + B_1y \leq C_1 \\ A_2x + B_2y \geq C_2 \\ A_3x + B_3y \leq C_3 \end{cases}$

The solution set for the system of inequalities is:

If the system represents the constraints in a modeling context, then the feasible region is:

Vocabulary

- **compound inequality in two variables**
- **feasible region**
- **solution set for the system of inequalities**
- solution to an equation (satisfies an equation)

Lesson Summary

In this lesson, we learned how to find the solution to a system of inequalities by finding the points in the coordinate plane that satisfy all of the inequalities simultaneously. We also used the solution to the system of inequalities for the *Pet Sitters* constraints to find the feasible region of viable options for Carlos and Clarita to consider when starting their business.

Unit 5, Lesson 7

More or Less

Jump Start

Graph the solution set for each of the following single-variable inequalities by labeling a number line and shading the solutions:

1. $2x + 3 \leq 7$
2. $2x + 3 \geq 7$
3. $2x + 3 < 7$
4. Since all of the coefficients are the same, what accounts for the difference in the solution sets?

Learning Focus

Solve systems of linear inequalities.

How do I interpret inequality signs, such as $>$, $<$, $\geq$, and $\leq$ when determining what to shade as a solution set to an inequality?

Open Up the Math: Launch, Explore, Discuss

Solve the following systems of inequalities:

1. Complete:

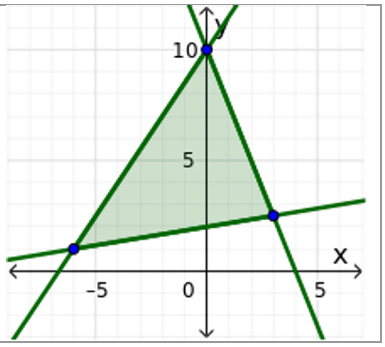
$\begin{cases} -5x + 3y \leq 45 \\ 2x + 3y > 24 \end{cases}$	

2. Complete:

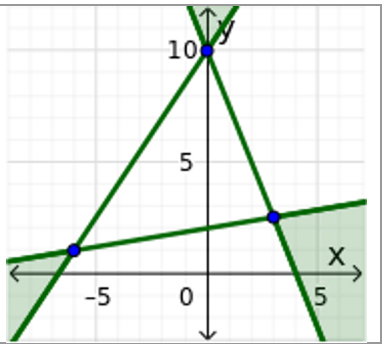
$\begin{cases} -10x + 6y \leq 90 \\ 6x + 9y > 36 \end{cases}$	

3. Is the point $(-3,10)$ a solution to the system in problem 1? Why or why not?

4.

Write the system of inequalities whose solution set is shown:	

5.

Amanda is examining Frank's work on problem 4, when she exclaims, "You have written all of your inequalities backwards. The solution set to your system would look like this." What do you think about Amanda's statement?	

Pause and Reflect

Carlos and Clarita have found a cat food that seems to appeal to even the most finicky of cats, *Figaro Flakes*. They want to mix it with a less expensive cat food, *Tabitha Tidbits*, to make an affordable, but tasty cat food.

Tabitha Tidbits contains 4 grams of protein and 6 grams of fat per scoop. *Figaro Flakes* contains 12 grams of protein and 4 grams of fat per scoop. Carlos wants to make a meal for cats that contains at least 8 grams of protein and no more than 6 grams of fat per scoop.

6. Write and solve a system of inequalities that Carlos can use to determine possible combinations of *Tabitha Tidbits* and *Figaro Flakes* that will satisfy both of these constraints.

7. Based on your work, suggest at least 3 different "recipes" using each type of cat food that meets Carlos's nutritional goals. For example, would 1 scoop of *Tabitha Tidbits* and $\frac{1}{2}$ scoop of *Figaro Flakes* be an acceptable meal?

Ready for More?

Write a system of inequalities whose solution set would be a region of the plane that forms some geometric shape, such as a square, a rhombus, a parallelogram, or a trapezoid.

Takeaways

Describe a procedure for modeling a context using a system of inequalities as constraints:

Lesson Summary

In this lesson, we learned how to differentiate between linear inequalities in two variables written as strict inequalities, inequalities with $<$ or $>$ signs, and other inequalities written with $\leq$ or $\geq$. We also practiced finding the solution set for systems of linear inequalities.

Unit 5, Lesson 8

Shopping for Cats and Dogs

Learning Focus

Develop intuitive strategies for solving systems of linear equations in standard form.

How can I use logical reasoning to solve for two unknown values when I am given two pieces of information about those values?

Open Up the Math: Launch, Explore, Discuss

Clarita is upset with Carlos because he has been buying cat and dog food without recording the price of each type of food in their accounting records. Instead, Carlos has just recorded the total price of each purchase, even though the total cost includes more than one type of food. Carlos is now trying to figure out the price of each type of food by reviewing some recent purchases.

As Carlos is examining the first set of purchases, he realizes that he can figure out the cost of the individual items just by reasoning about the numbers and the assumption that the price of each item remained the same for each shopping trip. Clarita is surprised that Carlos can find the individual prices without using tables, graphs, or equations.

See if you can reason about these shopping scenarios as well as Carlos by figuring out the cost of each item purchased without using tables, graphs, or equations.

1. One week Carlos bought 3 bags of *Tabitha Tidbits* and 4 bags of *Figaro Flakes* for $\$43.00$. The next week he bought 3 bags of *Tabitha Tidbits* and 6 bags of *Figaro Flakes* for $\$54.00$. Based on this information, figure out the price of one bag of each type of cat food. Explain your reasoning.

2. One week Carlos bought 2 bags of *Brutus Bites* and 3 bags of *Milo Munchies* for $\$42.50$. The next week he bought 5 bags of *Brutus Bites* and 6 bags of *Milo Munchies* for $\$94.25$. Based on this information, figure out the price of one bag of each type of dog food. Explain your reasoning.

3. Carlos purchased 6 dog leashes and 6 cat brushes for $\$45.00$ for Clarita to use while pampering the pets. Later in the summer he purchased 3 additional dog leashes and 2 cat brushes for $\$19.00$. Based on this information, figure out the price of each item. Explain your reasoning.

Pause and Reflect

4. One week Carlos purchased 2 boxes of cat treats and 3 boxes of dog treats for $\$18.50$. The next week, he bought 2 boxes of cat treats and 2 boxes of dog treats for $\$14.00$. The third week he bought 5 boxes of both cat and dog treats for $\$35.00$. Based on this information, figure out the price of each item. Explain your reasoning.

5. Carlos has noticed that because each of his purchases have been somewhat similar, it has been easy to figure out the cost of each item. However, his last set of receipts has him puzzled. One week he tried out cheaper brands of cat and dog food. On Monday he purchased 3 small bags of cat food and 5 small bags of dog food for $\$22.75$. Because he went through the small bags quite quickly, he had to return to the store on Thursday to buy 2 more small bags of cat food and 3 more small bags of dog food, which cost him $\$14.25$. Based on this information, figure out the price of each bag of the cheaper cat and dog food. Explain your reasoning.

Ready for More?

One week Carlos bought 2 packages of dog bones and 4 packages of cat treats for $\$18.50$. Because the finicky cats didn't like the cat treats, the next week Carlos returned 3 unopened packages of cat treats and bought 2 more packages of dog bones. After being refunded for the cat treats, Carlos only had to pay $\$1.00$ for his purchase. Based on this information, figure out the price of each item. Explain your reasoning.

Takeaways

Summarize the strategies you used to reason about the price of individual items in the context of purchasing different combinations of those items. What are some key ideas that seem helpful?

Lesson Summary

In this lesson, we developed a strategy for finding the price of individual items from knowing the total cost of purchasing different combinations of those items. We realized we could invent purchases that weren't actually made by doubling or tripling a known purchase or adding two purchases together. Finding two purchases in which the number of one of the items purchased was the same gave us insights into how to find the price of each individual item.

Unit 5, Lesson 9

Can You Get to the Point, Too?

Learning Focus

Solve systems of linear equations by eliminating one of the variables.

How do I use the logical reasoning for solving the scenarios in the previous task when the scenarios are represented with linear equations in standard form?

Open Up the Math: Launch, Explore, Discuss

In *Shopping for Cats and Dogs*, Carlos found a way to find the cost of individual items when given the purchase price of two different combinations of those items. He would like to make his strategy more efficient by writing it out using symbols and algebra. Help him formalize his strategy by doing the following:

- For each scenario in *Shopping for Cats and Dogs* write a **system of equations** to represent the two purchases.
- Show how your strategies for finding the cost of individual items could be represented by manipulating the equations in the system. Write out intermediate steps symbolically, so that someone else could follow your work.
- Once you find the price of one of the items in the combination, show how you would find the price of the other item.

1. One week Carlos bought 3 bags of *Tabitha Tidbits* and 4 bags of *Figaro Flakes* for $\$43.00$. The next week he bought 3 bags of *Tabitha Tidbits* and 6 bags of *Figaro Flakes* for $\$54.00$. Based on this information, figure out the price of one bag of each type of cat food. Explain your reasoning.

2. One week Carlos bought 2 bags of *Brutus Bites* and 3 bags of *Milo Munchies* for $\$42.50$. The next week he bought 5 bags of *Brutus Bites* and 6 bags of *Milo Munchies* for $\$94.25$. Based on this information, figure out the price of one bag of each type of dog food. Explain your reasoning.

3. Carlos purchased 6 dog leashes and 6 cat brushes for $\$45.00$ for Clarita to use while pampering the pets. Later in the summer he purchased 3 additional dog leashes and 2 cat brushes for $\$19.00$. Based on this information, figure out the price of each item. Explain your reasoning.

4. One week Carlos purchased 2 boxes of cat treats and 3 boxes of dog treats for $\$18.50$. The next week, he bought 2 box of cat treats and 2 boxes of dog treats for $\$14.00$. The third week he bought 5 boxes of both cat and dog treats for $\$35.00$. Based on this information, figure out the price of each item. Explain your reasoning.

5. Carlos has noticed that because each of his purchases has been somewhat similar, it has been easy to figure out the cost of each item. However, his last set of receipts has him puzzled. One week he tried out cheaper brands of cat and dog food. On Monday he purchased 3 small bags of cat food and 5 small bags of dog food for $\$22.75$. Because he went through the small bags quite quickly, he had to return to the store on Thursday to buy 2 more small bags of cat food and 3 more small bags of dog food, which cost him $\$14.25$. Based on this information, figure out the price of each bag of the cheaper cat and dog food. Explain your reasoning.

Pause and Reflect

While working on each of these problems, at some point you not only eliminated a variable, but you probably also eliminated an equation. To remind ourselves that we are working with a system of equations, it might be helpful to write out the solution process as a sequence of equivalent systems. To do so, we can obtain an equivalent system of equations by replacing one or both equations in the system using one of the following actions:

- Replace an equation in the system with a constant multiple of that equation.
- Replace an equation in the system with the sum or difference of the two equations.
- Replace an equation with the sum of that equation and a multiple of the other.

6. Solve the following problem by keeping track of both equations in the system each step along the way. That is, each step in your solution process will be a system of two linear equations. Write an explanation that explains how you changed the previous system to get the next system. The final system in the sequence will be $\begin{cases} d = \text{ a number} \\ c = \text{ a number} \end{cases}$

   Carlos purchased 4 dog ID tags and 12 cat ID tags for $\$36.00$ to keep track of the pet owner's contact information. Later in the summer he purchased 10 additional dog tags and 6 cat tags for $\$30.00$. Based on this information, figure out the price of each item.

System	Explanation
$\begin{cases} 4d + 12c = 36 \\ 10d + 6c = 30 \end{cases}$	Original system for the constraints.
$\begin{cases} 2d + 6c = 18 \\ 10d + 6c = 30 \end{cases}$	
$\begin{cases} 8d + 0c = 12 \\ 10d + 6c = 30 \end{cases}$	

Ready for More?

Writing out each system of equations reminded Carlos of his work with solving systems of equations graphically. Show how the cost of each item shows up graphically in each system in the solution process for problem 6. Record each system, and then sketch its graph on a separate grid. You may need fewer or more grids than are given, depending upon the number of systems in your solution process.

1. System: $\begin{cases} 4d + 12c = 36 \\ 10d + 6c = 30 \end{cases}$
2. System: $\begin{cases} 2d + 6c = 18 \\ 10d + 6c = 30 \end{cases}$
3. System: $\begin{cases} 8d + 0c = 12 \\ 10d + 6c = 30 \end{cases}$
4. System: $\begin{cases} d = 1.5 \\ 10d + 6c = 30 \end{cases}$
5. System: $\begin{cases} d = 1.5 \\ 6c = 15 \end{cases}$

6. System: $\begin{cases} d = 1.5 \\ c = 2.5 \end{cases}$

Takeaways

When solving equations, we write a sequence of equivalent equations until the solution to the equation is apparent.

Likewise, when solving systems of equations, we:

To create equivalent systems of equations we can:

Vocabulary

- **solve a system by elimination or substitution**

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we learned how to solve systems of equations by eliminating one of the variables. To do so, we had to think of equations as objects that can be added, subtracted, or multiplied by a scale factor. Each operation creates an equivalent system of equations, and if we are strategic, we can get a system of equations for which the solutions to the system are apparent.

Unit 5, Lesson 10

Taken Out of Context

Jump Start

Your teacher will distribute a set of cards to you and your partner.

The "begin here" card gives you a system of equations.

Put the remaining cards in the order you would use to find the solution to the system.

On each card you will need to write an additional piece of information: (1) the missing equation on the top of the card, or (2) the missing justification for what you did to transition from the previous card to the current one.

Learning Focus

Solve systems of equations.

Identify systems that have no solutions or an infinite number of solutions.

Do all systems of linear equations have a solution?

Can a system of linear equations have more than one solution?

What features of a context help me think about the nature of the solution?

Open Up the Math: Launch, Explore, Discuss

Write a shopping scenario similar to those in *Shopping for Cats and Dogs* to fit each of the following systems of equations. Then use the elimination of variables method you invented in the previous lesson to solve the system. Some of the systems may have interesting or unusual solutions. See if you can explain them in terms of the shopping scenarios you wrote.

If you can't find the solution to the system by elimination, try solving the system graphically or by substitution.

1. $\begin{cases} 3x + 4y = 23 \\ 5x + 3y = 31 \end{cases}$

2. $\begin{cases} 2x + 3y = 14 \\ 4x + 6y = 28 \end{cases}$

3. $\begin{cases} 3x + 2y = 20 \\ 9x + 6y = 36 \end{cases}$

4. $\begin{cases} 4x + 2y = 8 \\ 5x + 3y = 9 \end{cases}$

Pause and Reflect

5. Three of Carlos and Clarita's friends are purchasing school supplies at the bookstore. Stan buys a notebook, 3 packages of pencils, and 2 markers for $\$7.50$. Jan buys 2 notebooks, 6 packages of pencils, and 5 markers for

$15.50. Fran buys 1 notebook, 2 packages of pencils, and 2 markers for $6.25. How much do each of these 3 items cost?

The story context can be represented by this system of equations: $\begin{cases} n + 3p + 2m = 7.50 \\ 2n + 6p + 5m = 15.50 \\ n + 2p + 2m = 6.25 \end{cases}$

Explain in words or with symbols how you can use your intuitive reasoning about these purchases to find the price of each item.

Ready for More?

Create a linear system with three variables, x, y, and z, by first deciding what you want the values of each variable to be and then writing three equations with random choices for the coefficients. Calculate the correct sums of the three equations using the known values of the variables.

Trade your system with a partner and see if you can solve the system that was created for you, and if your partner can solve the system that was created for them.

What features of a system make this easy? What makes it hard?

Takeaways

Describe the following systems of equations:

A dependent system:

An inconsistent system:

Finding a unique solution for the variables in an independent system of equations requires:

Vocabulary

- **systems: inconsistent / independent**

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we learned that systems of linear equations can have infinitely many solutions, no solutions, or one solution, and we learned how to identify which type of solution to expect by examining the coefficients of the linear equations in the system when they are written in standard form. We also extended the process of solving systems to a system of linear equations with three unknowns.