Unit 4, Lesson 1

Cafeteria Actions and Reactions

Jump Start

Evaluate the following expression for $x = 3$. Show each step of your work so you can review it with a neighbor.

$$\frac{4(2x + 3) + 4}{8}$$

Learning Focus

Solve multi-step linear equations using inverse operations.

How does connecting the operations in an equation to an action in a story help me develop and justify my solution process for solving the equation?

How does the structure of an equation give me clues about how to solve it?

Open Up the Math: Launch, Explore, Discuss

Elvira, the cafeteria manager, has just received a shipment of new trays with the school logo prominently displayed in the middle of each tray. After unloading 4 cartons of trays in the pizza line, she realizes that students are arriving for lunch and she will have to wait until lunch is over before unloading the remaining cartons. The new trays are very popular, and in just a couple of minutes, 24 students have passed through the pizza line and are showing off the school logo on the trays. At this time, Elvira decides to divide the remaining trays in the pizza line into 3 equal groups so she can also place some in the salad line and the sandwich line, hoping to attract students to the other lines. After doing so, she realizes that each of the 3 serving lines has only 12 of the new trays.

"That's not many trays for each line. I wonder how many trays there were in each of the cartons I unloaded?"

1. Help the cafeteria manager answer her question using the data in the story about each of the actions she took. Explain how you arrived at your solution.

Elvira is interested in collecting data about how many students use each table during lunch. She has recorded some data on sticky notes to analyze later. Here are the notes she has recorded:

- Some students are sitting at the front table. (I got distracted by an incident in the back of the lunchroom, and forgot to record how many students.)
- Each of the students at the front table has been joined by a friend, doubling the number of students at the table.
- Four more students have just taken seats with the students at the front table.
- The students at the front table separated into three equal-sized groups and then two groups left, leaving only one-third of the students at the table.
- As the lunch period ends, there are still 12 students seated at the front table.

Elvira is wondering how many students were sitting at the front table when she wrote her first note. Unfortunately, she is not sure what order the middle three sticky notes were recorded in since they got stuck together in random order. She is wondering if it matters.

2. Does it matter which order the notes were recorded in? Determine how many students were originally sitting at the front table based on the sequence of notes that appears above. Then, rearrange the middle three notes in different orders and determine what the new order implies about the number of students seated at the front table at the beginning.

Pause and Reflect

3. Here are three different equations that could be written based on a particular sequence of notes. Examine each equation, and then list the order of the five notes that is represented by each equation. Find the solution for each equation.

   - $\frac{2(x+4)}{3} = 12$
   - $2\left(\frac{x}{3} + 4\right) = 12$
   - $\frac{2x+4}{3} = 12$

Ready for More?

1. Create your own story context with a sequence of actions that can be represented symbolically with an equation. Then, create a set of sticky note statements for your situation. Rearrange your notes in ways that can be represented by a variety of different equations, even though the actions remain the same, then record your new equations with the corresponding stories.
2. Are there arrangements of the notes that produce different answers? If so, list the equations.
3. Can you find different arrangements of the notes that produce the same answer? Why might that be so? List the equations.

Takeaways

Today, we used operations of arithmetic to represent actions in a story context. We used grouping symbols and order of operations to write an equation to model a story context, and then used the story context to help us justify a process for solving the equation.

In general, we learned that when solving a linear equation, we un-do operations in the equation by _____.

The order for applying inverse operations is _____.

We can justify each step in the equation-solving process by using properties of equality and properties of operations, such as:

The Distributive Property: _____

The Addition Property of Equality: _____

The Subtraction Property of Equality: _____

The Multiplication Property of Equality: _____

The Division Property of Equality: _____

Vocabulary

- distributive property of multiplication over addition

- **inverse operation**
- **multi-step equation**
- **properties of equality**

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we learned how to solve multi-step equations in the context of using operations to represent actions in a story. As we observed how to "un-do" the actions of the story, we developed a strategy for solving the equation by using inverse operations. The order in which inverse operations are applied when solving an equation matters, and we learned how to pay attention to the structure of the equation for clues to the order in which we should use inverse operations.

Unit 4, Lesson 2

Elvira's Equations

Jump Start

Quantities are attributes that can be counted or measured, such as a dozen eggs, 25 miles, or 60 minutes. Notice that quantities always include a unit of measure: eggs, miles, minutes. Some quantities are derived from combining other measurements, such as miles per gallon, or feet per minute.

1. Describe how you would measure the fuel efficiency of your family car in miles per gallon.
2. Describe how you would measure the speed of a baseball in feet per second.
3. Describe how you would measure the area of a rectangular room in square feet.
4. Describe how you would calculate the revenue collected by a roadside fruit stand if they sell 20 bags of oranges for $\$5$ per bag.

Learning Focus

Use units to interpret and solve equations that contain primarily variables that represent quantities, such as a formula.

What does it mean to solve an equation that contains multiple variables?

How can I use descriptions and units associated with variables to guide my thinking about solving such equations?

Open Up the Math: Launch, Explore, Discuss

Elvira, the cafeteria manager, likes to keep track of the things she can count or measure in the cafeteria. She hopes this will help her improve the efficiency of the cafeteria. To remind herself to keep track of important quantities, she has made a table of variables and descriptions of the things she wants to record. Here is a table of things she has decided to keep track of.

1. Determine the units that fit in the third column of the table for each of the variables whose descriptions are given.

Symbol	Meaning (description of what the symbol means in context)	Units (what is counted or measured)
S	Number of students who buy lunch in the salad line	
W	Number of students who buy lunch in the sandwich line	
P	Number of students who buy lunch in the pizza line	
F	Number of food servers in the cafeteria	
M_T	Number of minutes it takes to serve lunch to all students	
C	Number of classes in the school	
P_L	Price per lunch	
A		
R		
T		
D_F		
M		

Elvira has written the following equation to describe a cafeteria relationship that seems meaningful to her. She has introduced a new variable, A, to describe this relationship.

$$A = \frac{S + W + P}{C}$$

2. What does A represent in terms of the cafeteria? Record this information in the table.

3. Using what you know about manipulating equations, solve this equation for S. Your solution will be of the form $S =$ an expression written in terms of the variables A, C, W, and P.

4. Does your expression for S make sense in terms of the meanings of the other variables? Explain why or why not.

Here is another one of Elvira's equations.

$$R = P_L(S + W + P)$$

5. What does R represent in terms of the cafeteria? Record this information in the table.

6. Using what you know about manipulating equations, solve this equation for P_L.

7. Does your expression for P_L make sense in terms of the meanings of the other variables? Explain why or why not.

8. Elvira notices that she uses the expression $S + W + P$ a lot in writing other expressions. She decides to represent this expression using the variable T so that $T = S + W + P$. What does T represent in terms of the cafeteria? Record this information in the table.

Elvira is having a meeting with the staff members who work in the lunchroom. She has created a couple of new equations for the food servers.

$D_F = \frac{T \cdot P_L}{F}$	$M = \frac{M_T}{T}$

9.

   a. What does D_F represent in terms of the cafeteria? Record this information in the table.

   b. Solve this equation for P_L. Describe why your solution makes sense in terms of the other variables.

Pause and Reflect

10. $M = \frac{M_T}{T}$

   a. What does M represent in terms of the cafeteria? Record this information in the table.

   b. Solve this equation for T. Describe why your solution makes sense in terms of the other variables.

11. One of the staff members suggests that they need to write expressions for each of the following sentences. Using the variables in the table, what would these expressions look like?

   a. The average number of students served each minute.

   b. The average number of minutes students wait in the pizza line.

Ready for More?

Use the variables in the table to create other meaningful expressions. What might be the longest expression (in terms of number of variables required) that can be written using these variables?

Takeaways

Equations that involve multiple variables are called literal equations. If the variables in the equation represent meaningful quantities, literal equations are also referred to as formulas.

To solve a literal equation for one of its variables, I can

If the variables in the equation represent meaningful quantities, I can

When determining the meaning of an expression that contains multiple quantities, I need to attend to the units and operations. Some strategies for doing so include

Vocabulary

- **formula**
- inverse operation
- **literal equation**
- profit
- quantity
- reciprocal
- revenue
- units

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we learned how to solve literal equations for one of its variables using inverse operations with both variables and numbers. Literal equations are formulas for describing the relationships between multiple quantities. Interpreting the meaning of expressions involving quantities in terms of their units can be a tool for checking our algebraic work while solving equations.

Unit 4, Lesson 3

Solving Equations Literally

Learning Focus

Compare strategies for solving linear equations and literal equations.

How is solving an equation with one variable similar to the work of solving an equation with more than one variable?

Open Up the Math: Launch, Explore, Discuss

Notice and Wonder

Solve each of the following equations for x:

1. $\frac{3x+2}{5} = 7$
2. $\frac{3x+2y}{5} = 7$
3. $\frac{4x}{3} - 5 = 11$
4. $\frac{4x}{3} - 5y = 11$
5. $\frac{2}{5}(x+3) = 6$
6. $\frac{2}{5}(x+y) = 6$
7. $2(3x+4) = 4x+12$
8. $2(3x+4y) = 4x+12y$

Write a verbal description for each step of the equation-solving process used to solve the following equations for x. Your description should include statements about how you know what to do next. For example, you might write, "First I _____. The reason I can do this is because of _____."

9. $\frac{ax+b}{c} - d = e$
10. $r \cdot \sqrt{\frac{mx}{n} + s} = t$

Ready for More?

Work with a partner to revise and refine your justifications for problems 9 and 10. For example, what properties might you use to convince yourself that $\frac{1}{r} \cdot r \cdot \sqrt{\frac{mx}{n} + s} = \frac{1}{r} \cdot t$ is equivalent to $\sqrt{\frac{mx}{n} + s} = \frac{t}{r}$?

Takeaways

Today I gained some new insights into the process for solving equations, including:

When solving literal equations, we may need to change the form of the expressions involving variables and operations. The following properties of operations justify how we might rewrite these expressions:

Properties of Addition			
Additive Identity	Additive Inverse	Commutative	Associative
Properties of Multiplication			
Multiplicative Identity	Multiplicative Inverse	Commutative	Associative
The Distributive Property			

Vocabulary

- associative property of addition or multiplication
- commutative property of addition or multiplication
- distributive property of multiplication over addition
- **identity: additive, multiplicative**
- **inverse: additive, multiplicative**
- properties of operations for numbers in the rational, real, or complex number systems

Bold terms are new in this lesson.

Lesson Summary

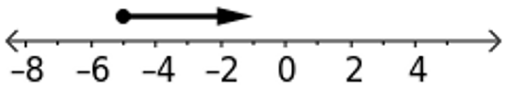
In this lesson, we compared strategies for solving linear equations and literal equations and found that the processes for solving each are similar. We solve both types of equations by using inverse operations in the reverse order from the order used when evaluating the expression that involves the variable we are solving for. However, the answer to a linear equation is a number, while the answer to a literal equation is a variable or an expression. Sometimes we have to combine like terms, particularly if expressions containing the same variable occur on both sides of the equations. Properties of operations and properties of equality guide our thinking when solving equations and help us justify each step in our equation-solving process.

Unit 4, Lesson 4

Greater Than?

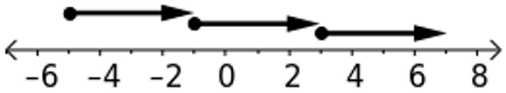
Jump Start

Which One Doesn't Belong? Examine each of the models below and determine which one is different from the rest. Be prepared to justify your answer.

A. 

B. $x + 4$

C. 

D. 

Reason:

Learning Focus

Reason about inequalities.

Justify properties of inequalities.

Use inequality notation.

What are the properties of inequalities? Are they different from the properties of equations?

Open Up the Math: Launch, Explore, Discuss

Given a mathematical statement and two expressions, decide which of the two expressions is greater, if the expressions are equal, or if the relationship cannot be determined from the statement. Write an equation or inequality that shows your answer and explain why your answer is correct. Watch out—this gets tricky!

Example:

Statement: $x = 8$	Which is greater? $x + 5$ or $3x + 2$

$3x + 2$ is greater. $3x + 2 > x + 5$. Because if $x = 8$, $3x + 2 = 26$, $x + 5 = 13$, and $26 > 13$.

1.

Statement: n is an integer.	Which is greater? n or $-n$

Try it yourself:

2.

Statement: $x > y$	Which is greater? $x + a$ or $y + a$

3.

Statement: $x > y$	Which is greater? $x - a$ or $y - a$

4.

Statement: $5 > 4$	Which is greater? $5x$ or $4x$

5.

Statement: $5 > 4$	Which is greater? $\frac{5}{x}$ or $\frac{4}{x}$

6.

Statement: $y < x$	Which is greater? $x - y$ or $y - x$

7.

Statement: $2x - 3 > 7$	Which is greater? 5 or x

8.

Statement: $10 - 2x < 6$	Which is greater? x or 2

9.

Statement: $4x \leq 0$	Which is greater? 1 or x

10.

Statement: $0 < x < 10$ and $0 < y < 12$	Which is greater? x or y

11.

Statement: $3^{n+2} \geq 27$	Which is greater? n or 1

Ready for More?

Here is one more problem that will make you dig a little deeper. See what you can do!

Statement: $a > 0, b < 0$

Which is greater? ab or $\frac{a}{b}$?

Takeaways

Properties of Inequalities

Addition Property:

Subtraction Property:

Multiplication Property:

Division Property:

Vocabulary

- inequality
- **properties of inequality**

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we reasoned about inequalities to compare algebraic expressions. We found and justified the addition, subtraction, multiplication, and division properties of inequalities.

Unit 4, Lesson 5

May I Have More, Please?

Jump Start

Determine which inequality matches each phrase.

- $x > 7$
- $x \geq 7$
- $x < 7$
- $x \leq 7$

1. No more than 7
2. A minimum of 7
3. More than 7
4. No less than 7
5. At least 7
6. A maximum of 7
7. Cannot exceed 7
8. Fewer than 7

Learning Focus

Write and solve inequalities to model real situations.

Write solutions to inequalities using set builder and interval notation.

How can inequalities be used to find solutions to real problem situations?

Open Up the Math: Launch, Explore, Discuss

Elvira, the cafeteria manager, needs to be careful with her spending, and she manages the cafeteria budget so that they can serve the best food at the lowest cost. To do this, Elvira keeps good records and analyzes all her budgets.

1. Elvira's cafeteria has those cute little cartons of milk that are typical of school lunch. The milk supplier charges $\$0.35$ per carton of milk, in addition to a delivery charge of $\$75$. What is the maximum number of milk cartons that Elvira can buy if she has budgeted $\$500$ for milk?

    a. Write and solve an inequality that models this situation.

    b. Describe in words the quantities that would work in this situation.

c. Write your answer in both interval and set notation.

2. Students love to put ranch dressing on everything, so Elvira needs to keep plenty in stock. The students eat about 2.25 gallons of ranch each day! Elvira started the school year with 130 gallons of ranch dressing. She needs to have at least 20 gallons left when she reorders to have enough in stock until the new order comes. For how many days will her ranch dressing supply last before she needs to reorder?

   a. Write and solve an inequality that models this situation.

   b. Describe in words the quantities that would work in this situation.

   c. Write your answer in both interval and set notation.

3. The prices on many of the cafeteria foods change during the year. Elvira finds that she has ordered veggie burgers four times and paid $\$78$, $\$72$, $\$87$, and $\$90$ on the orders. To stay within her budget, Elvira needs to be sure that the average order of veggie burgers is not more than $\$82$. How much can she spend on the fifth order to keep the average order within her budget?

   a. Write and solve an inequality that models this situation.

   b. Describe in words the quantities that would work in this situation.

   c. Write your answer in both interval and set notation.

4. Elvira can purchase ready-made pizzas for $\$14.50$ each. If she makes them in the cafeteria, she can spend $\$44.20$ on ingredients and $\$6.25$ per pizza on labor. For how many pizzas is it cheaper for the cafeteria to make the pizzas themselves rather than buy them ready-made?

   a. Write and solve an inequality that models this situation.

   b. Describe in words the quantities that would work in this situation.

   c. Write your answer in both interval and set notation.

5. Elvira is comparing prices between two different suppliers of fresh lettuce. Val's Veggies charges $\$250$ for delivery plus $\$1.50$ per bag of lettuce. Sally's Salads charges $\$100$ for delivery plus $\$4.00$ per bag of lettuce. How many bags of lettuce must be purchased for Val's Veggies to be the cheaper option?

   a. Write and solve an inequality that models this situation.

   b. Describe in words the quantities that would work in this situation.

   c. Write your answer in both interval and set notation.

6. Each student who buys a school lunch pays $\$2.75$. The cafeteria typically brings in between $\$1,168.75$ and $\$1,438.25$ each day. How many students does the cafeteria usually serve in a day?

   a. Model this situation using an inequality.

   b. Describe in words the quantities that would work in this situation.

   c. Write your answer in both interval and set notation.

Ready for More?

River A and River B combine together to form River C. The flow rate of the River C is less than or equal to the sum of the flow rates of the Rivers A and B, but it is greater than or equal to either of the individual flow rates of Rivers A and B. One of the three rivers flows at a rate of 52 gallons per minute. Another river flows at a rate of 38 gallons per minute.

1. Find the minimum flow rate, f, of the third river.
2. Find the maximum flow rate, f, of the third river.
3. Write an inequality that describes the possible flow rates, f, of the third river.

Takeaways

Using properties of inequalities to solve inequalities:

Adding Notation, Vocabulary, and Conventions

Compound inequality:

Examples:

Vocabulary

- **compound inequality in one variable**

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we wrote inequalities to model contexts that had a range of solutions. We used the properties of inequalities to solve the inequalities and found that solving inequalities is very similar to solving equations, but we must be careful when multiplying or dividing by a negative number to reverse the inequality sign. We used interval and set notation to write solutions and learned that many of the solutions that we write are compound inequalities.

Unit 4, Lesson 6

Taking Sides

Learning Focus

Understand similarities and differences in solving equations and inequalities.

Learn to avoid common errors and misunderstandings about inequalities.

What are some of the common misconceptions of inequalities?

How does having a deep understanding of what inequalities mean help to avoid errors?

Can an inequality have no solutions?

Open Up the Math: Launch, Explore, Discuss

Joaquin and Serena work together productively in their math class. They both contribute their thinking and when they disagree, they both give their reasons and decide together who is right. In their math class right now, they are working on inequalities. Recently, they had a discussion that went something like this:

Joaquin: The problem says that "6 less than a number is greater than 4." I think that we should just follow the words and write: $6 - n > 4$.

Serena: I don't think that works because if n is 20 and you do 6 less than that you get $20 - 6 = 14$. I think we should write $n - 6 > 4$.

Joaquin: Oh, you're right. Then it makes sense that the solution will be $n > 10$, which means we can choose any number greater than 10.

The situations below are a few more of the disagreements and questions that Joaquin and Serena have. Your job is to decide how to answer their questions, decide who is right, and give a mathematical explanation of your reasoning.

1. Joaquin and Serena are assigned to graph the inequality $x \geq -7$.

   Joaquin thinks the graph should have an open dot at -7.

   Serena thinks the graph should have a closed dot at -7.

   Explain who is correct and why.

2. Joaquin and Serena are looking at the problem $3x + 1 > 0$.

   Serena says that the inequality is always true because multiplying a number by 3 and then adding 1 to it makes the number greater than 0.

   Is she right? Explain why or why not.

3. The word problem that Joaquin and Serena are working on says, "4 greater than x."

   Joaquin says that they should write: $4 > x$

   Serena says they should write: $4 + x$

   Explain who is correct and why.

4. Joaquin is thinking hard about equations and inequalities and comes up with this idea:

   If $45 + 47 = t$, then $t = 45 + 47$.

   So, if $45 + 47 < t$, then $t < 45 + 47$.

   Is he right? Explain why or why not.

5. Joaquin's question in problem 4 made Serena think about other similarities and differences in equations and inequalities. Serena wonders about the equation $-\frac{x}{3} = 4$ and the inequality $-\frac{x}{3} > 4$. Explain to Serena ways that solving these two problems are alike and ways that they are different. How are the solutions to the problems alike and different?

6.

   a. Joaquin solved $-15q \leq 135$ by adding 15 to each side of the inequality. Serena said that he was wrong. Who do you think is right and why?

   b. Joaquin's solution was $q \leq 150$. He checked his work by substituting 150 for q in the original inequality. Does this prove that Joaquin is right? Explain why or why not.

   c. Joaquin is still skeptical and believes he is right. Find a number that satisfies his solution but does not satisfy the original inequality.

7. Serena is checking her work with Joaquin and finds that they disagree on a problem. Here's what Serena wrote:

   $3x + 3 \leq -2x + 53x \leq -2x + 2x \leq 2$

   Is she right? Explain why or why not.

8. Joaquin and Serena are having trouble solving $-4(3m - 1) \geq 2(m + 3)$.

   Explain how they should solve the inequality, showing all the necessary steps and identifying the properties you would use.

9. Joaquin and Serena know that some equations are true for any value of the variable and some equations are never true, no matter what value is chosen for the variable. They are wondering about inequalities. What could you tell them about the following inequalities? Do they have solutions? What are they? How would you graph their solutions on a number line?

   a. $4s + 6 \geq 6 + 4s$

   b. $3r + 5 > 3r - 2$

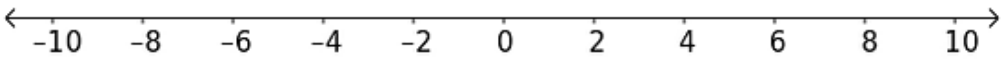
   c. $4(n + 1) < 4n - 3$

10. The partners are given the literal inequality $ax + b > c$ to solve for x. Joaquin says that he will solve it just like an equation. Serena says that he needs to be careful because if a is a negative number, the solution will be different. What do you say? What are the solutions for the inequality?

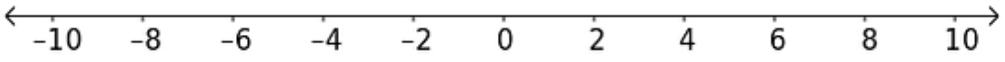
Ready for More?

Use your reasoning skills to solve the following inequalities. Write your solutions in both interval and set notation and graph on the number line.

1. $|x| < 5$



2. $|x| \geq 5$



Takeaways

Similarities in Solving Inequalities and Equations:

Differences in Solving Inequalities and Equations:

Lesson Summary

In this lesson, we examined common mistakes and misconceptions about inequalities. We analyzed the similarities and differences in solving equations and inequalities, determining which properties applied to both and which properties were different for inequalities.