Unit 3, Lesson 1

Getting Ready for a Pool Party

Learning Focus

Graph a function to model a situation.

Interpret and identify key features of the graph.

How can we create a graph without equations or points being given?

How does a graph tell a story?

How do I describe key features of a graph?

Open Up the Math: Launch, Explore, Discuss

Sylvia has a small pool full of water that needs to be emptied and cleaned, then refilled for a pool party. During the process of getting the pool ready, Sylvia did all of the following activities, each during a different time interval.

Removed water with a single bucket	Filled the pool with a hose (same rate as emptying pool)
Drained water with a hose (same rate as filling pool)	Cleaned the empty pool
Sylvia and her two friends removed water with her three buckets	Took a break

1. Create a story of Sylvia's process for emptying, cleaning, and filling the pool. Number the activities given in the table 1–6 to indicate the order in which they occurred in your story.

2. Sketch a possible graph showing the height of the water level in the pool over time. Be sure to include all of the activities Sylvia did to prepare the pool for the party. Remember that only one activity happened at a time. Think carefully about how each section of your graph will look, labeling where each activity occurs.

3. Does your graph represent a function? Why or why not? Would all graphs created for this situation represent a function?

Ready for More?

Write a story context that has one interval of increase, one interval of decrease, and one interval when the rate of change is 0, in any order. Then, graph your story.

Takeaways

A relationship is a function if and only if:

Adding Notation, Vocabulary, and Conventions

Key features of functions:

Vocabulary

- **continuous function/discontinuous function**
- discrete function
- domain
- function
- **interval of increase or decrease**
- **maximum / minimum**
- **range of a function**
- rate of change
- **x-intercept**
- y-intercept

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we created graphs to model a story context. We learned the mathematical words to describe the key features of functions. These key features are used to analyze and compare functions.

Unit 3, Lesson 2

Floating Down the River

Learning Focus

Identify key features of functions.

Use key features of functions to analyze tables and graphs.

How do I describe key features of a function?

Open Up the Math: Launch, Explore, Discuss

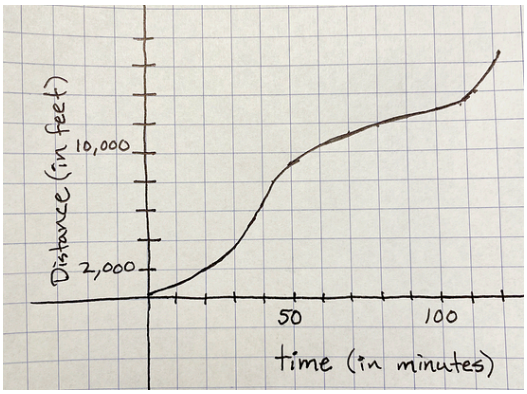
Part I:

Alonzo, Maria, and Sierra were floating in inner tubes down a river, enjoying their day. Alonzo noticed that sometimes the water was much deeper in some places than in others. Maria noticed there were times they seemed to be moving faster than at other times. Sierra laughed and said "Math is everywhere!" To learn more about the river, Alonzo and Maria collected data throughout the trip. Here is Alonzo's data table about the depth of the water as they floated along.

Time (in minutes)	0	10	20	30	40	50	60	70	80	90	100	110	120
Depth (in feet)	4	6	8	10	6	5	4	5	7	12	9	6.5	5

1. Use the data that Alonzo collected in the table to interpret the key features of this relationship.

Maria created a graph by collecting data on a GPS unit that provided the distance she had traveled over a period of time.



2. Describe the key features of the relationship shown in Maria's graph. Include the intervals of increase and decrease, domain, range, maximum, minimum, and intercepts.

Part II: Interpreting data

3. Sierra looked at the data collected by her two friends and made several of her own observations. Explain why you either agree or disagree with each observation made. Give a reason for why you agree or disagree.

a. The depth of the water increases and decreases throughout the 120 minutes of floating down the river.

b. The distance traveled is always increasing.

c. The distance traveled is a function of time.

d. The distance traveled is greatest during the last ten minutes of the trip than during any other ten-minute interval of time.

e. The domain of the distance/time graph is all real numbers.

f. The y-intercept of the depth of water over time function is $(0,0)$.

g. The distance traveled increases and decreases over time.

h. The depth of the water is never 11 feet.

i. The range of the distance/time graph is from $[0,15000]$.

j. The domain of the depth of water with respect to time is from $[0,120]$.

k. The range of the depth of water over time is from $[4,5]$.

l. The distance/ time graph has no maximum value.

m. The depth of water reached a maximum at 30 minutes.

Ready for More?

Use the data given in the table and the graph to graph the relationship between distance (independent variable) and depth (output variable) during the float trip and interpret the meaning. Create statements like Sierra's related to features of the function. Make three statements that are true and one statement that is false.

Takeaways

Strategies for Finding Features of Functions:

Adding Notation, Vocabulary, and Conventions

In words:	As an inequality:	In interval notation:

Vocabulary

- **interval notation**
- set

Lesson Summary

In the lesson we learned how to find features of functions in a table and clarified what each feature describes. We learned to use interval notation to write domains, ranges, and intervals of increase and decrease for continuous functions.

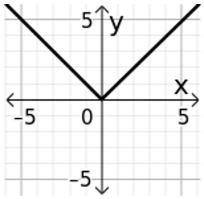
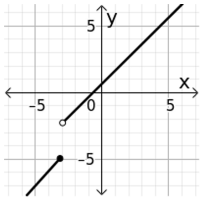
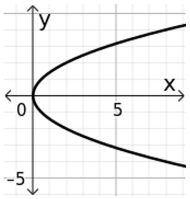
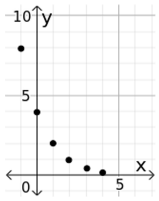
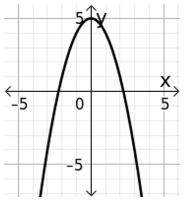
Unit 3, Lesson 3

Function Junction

Jump Start

Which One Doesn't Belong?

Use the mathematical features to analyze each relationship and choose which graph doesn't belong with the others. Explain your reasoning with mathematical vocabulary.

<table>
<tr>
<td>a. 
</td>
<td>b. 
</td>
<td>c. 
</td>
</tr>
<tr>
<td>d. 
</td>
<td>e. 
</td>
<td></td>
</tr>
</table>

Reason:

Learning Focus

Become efficient in identifying key features of functions in various representations.

Describe domain, range, and intervals of increase and decrease using appropriate notation.

How do I choose between interval and set builder notation for domains and ranges?

How can I use the relationship between features of functions to help me be more efficient in writing features?

How can I tell if a maximum or minimum is relative or absolute?

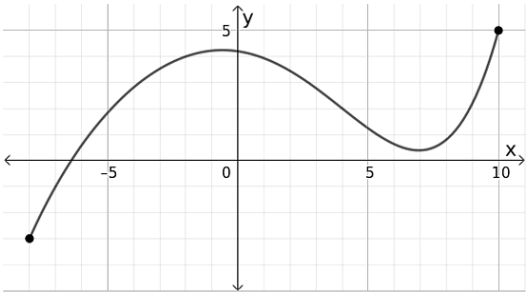
Open Up the Math: Launch, Explore, Discuss

Analyze each function to find the key features. Write each feature using appropriate mathematical notation.

1. The table represents a discrete function defined on the interval $[1,5]$.

n	$f(n)$
1	5
2	10
3	40
4	80
5	160

Domain: _____ Range: _____ Maximum: _____ Minimum: _____	Intercept(s): _____ Interval(s) of increase: _____ Interval(s) of decrease: _____

2. 

Domain: _____ Range: _____ Maximum: _____ Minimum: _____	Intercept(s): _____ Interval(s) of increase: _____ Interval(s) of decrease: _____ Function (Yes or No): _____

3. $g(x) = 3^x$

Graph the function, then determine the key features.

Domain: _____ Range: _____ Maximum: _____ Minimum: _____	Intercept(s): _____ Interval(s) of increase: _____ Interval(s) of decrease: _____

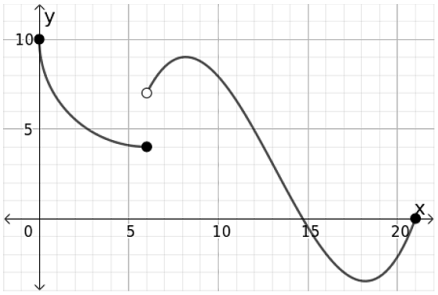
4. Marcus bought a $\$900$ couch on a six-month, interest free payment plan. He pays $\$50$ each week on the loan. Describe the key features of the relationship between the number of weeks and the amount owed on the loan.

Domain: _____ Range: _____ Maximum: _____ Minimum: _____ Intercept(s): _____	Interval(s) of increase: _____ Interval(s) of decrease: _____ Function (Yes or No): _____ Discrete/Continuous/Discontinuous: _____

5. The table represents a continuous function defined on the interval $[0,6]$.

x	$f(x)$
0	2
1	-3
2	0
3	2
4	6
5	12
6	20

Domain: _____ Range: _____ Maximum: _____ Minimum: _____	Intercept(s): _____ Interval(s) of increase: _____ Interval(s) of decrease: _____

6. 

Domain: _____ Range: _____ Maximum: _____ Minimum: _____	Intercept(s): _____ Interval(s) of increase: _____ Interval(s) of decrease: _____ Function (Yes or No): _____ Discrete/Continuous/Discontinuous: _____

7. Describe the key features of the relationship between the number of hours of daylight and the day of the year in your town. Consider January 1 as day 1, that the longest day of the year is day 173 (about June 22), and the shortest day of the year is day 356 (about December 23). You may need to make some reasonable estimates for the number of daylight hours.

Domain: _____ Range: _____ Maximum: _____ Minimum: _____ Intercept(s): _____	Interval(s) of increase: _____ Interval(s) of decrease: _____ Function (Yes or No): _____ Discrete/Continuous/Discontinuous: _____

8. $f(x) = -2x + 4$, when $x \geq 0$

   Graph the function, then determine the key features.

Domain: _____ Range: _____ Maximum: _____ Minimum: _____	Intercept(s): _____ Interval(s) of increase: _____ Interval(s) of decrease: _____

Ready for More?

Draw a graph of a function with the following features:

- Increases on the intervals $(-5,3) \cup (7, \infty)$
- Decreases on the intervals $(-\infty, -5) \cup (3,7)$
- Has a relative maximum of 4
- Has a relative minimum of -4 and another of -3
- Is continuous
- Contains the point $(0,2)$

Takeaways

Helpful ideas for finding and writing features of functions.

Lesson Summary

In this lesson, we worked on becoming fluent, flexible, and accurate in identifying and writing the key features of functions. We learned that domain and range can both be written as lists in set builder notation. We learned to identify features from context and to use graphs to help visualize the features.

Unit 3, Lesson 4

Water Wonderland

Learning Focus

Interpret the graphs and equations of functions.

Write equations for the graph of functions.

Combine two linear functions.

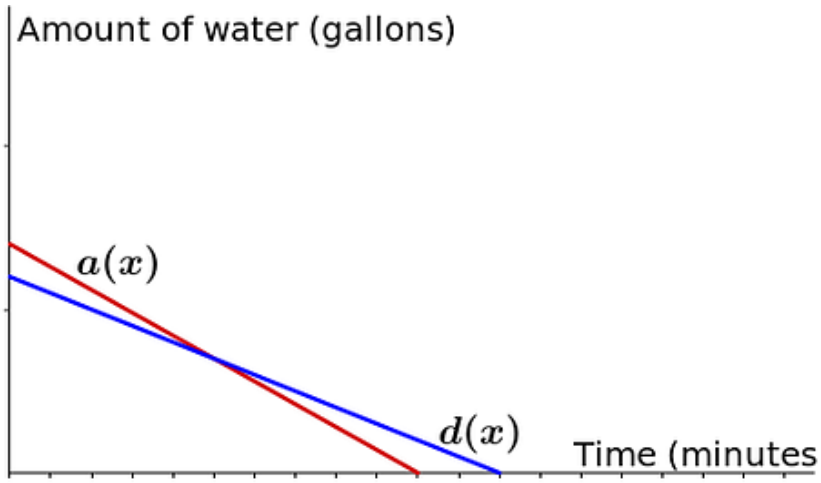
How does function notation connect to a graph?

What connections can be made with function notation and a story context?

How do I combine functions graphically and algebraically?

Open Up the Math: Launch, Explore, Discuss

Aly and Dayne work at a water park and are required to drain the water at the end of each month for the ride they supervise. Each uses a pump to remove the water from the small pool at the bottom of their ride. The graph given represents the amount of water in Aly's pool, $a(x)$, and Dayne's pool, $d(x)$, over time.



1. Make as many observations as possible with the information given in the graph.

Dayne figured out that the pump he uses drains water at a rate of $1,000$ gallons per minute and takes 24 minutes to drain.

2. Write the equation to represent the draining of Dayne's pool, $d(x)$. What does each part of the equation mean?
3. Based on this new information, correctly label the graph shown.
4. What values of x make sense in this situation? Write the domain of the function that represents the amount of water in Dayne's pool.
5. What output values make sense in this situation? Write the range of the function that represents the amount of water in Dayne's pool.

6. Write the equation that represents the draining of Aly's pool, $a(x)$. State the domain and range for the function, $a(x)$.

   Equation:

Domain:	Range:

Based on the graph and corresponding equations for each pool, answer the following questions.

7. When is $a(x) = d(x)$? What does this mean?

8. Find $a(5)$. What does this mean?

9. If $d(x) = 2000$, then $x =$ _____. What does this mean?

10. When is $a(x) > d(x)$? What does this mean?

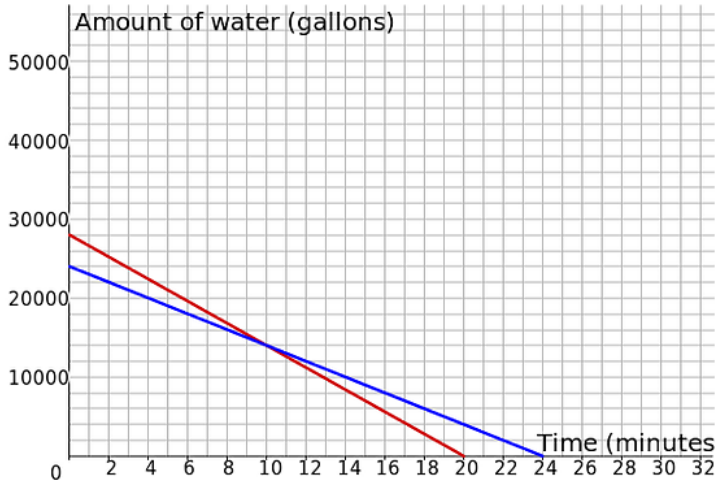
This month, Aly and Dayne decided to work together by putting both pumps in the pool at the same time to drain their pools and created the equation:

$$g(x) = a(x) + d(x)$$

When they wrote their equation, Aly and Dayne decided that they would only model the time when the pumps were in the pool so they didn't have to worry about the short time that it took to move the pumps from one pool to the next.

11. What does $g(x)$ represent?

12. Graph $g(x)$ on the same axes as $a(x)$ and $d(x)$.



13. Write the equation for the function $g(x)$ using the graph you created. Compare this equation to the algebraic representation for finding the sum of the equations for $a(x)$ and $d(x)$.

14. Should the algebraic equation of $g(x)$ be the same as the algebraic function created from the graph? Why or why not?

15. Use both the graphical and the algebraic representation to describe key features of and explain what each feature means (each intercept, domain and range for this situation and for the equation, maxima and minima, whether or not is a function, etc.)

Domain: What it means in this context: Range: What it means in this context: Interval(s) of increase: What it means in this context: Interval(s) of decrease: What it means in this context:	y-intercept: What it means in this context: x-intercept: What it means in this context: Rate of change: What it means in this context: Continuous? Why? Function? Why?

16. Explain why adding the two values of the y-intercepts together in $a(x)$ and $d(x)$ can be used to find the y-intercept in $g(x)$.

17. Can a similar method be used to find the x-intercept of $g(x)$? Explain.

Ready for More?

Fletcher, another employee, decided to start draining the pools before Aly and Dayne showed up for work. When Aly and Dayne arrived, there were already 5,000 fewer gallons of water to be drained. How does this change the equation for the amount of water Aly and Dayne must drain? How does this change the graph?

Takeaways

Function vocabulary and notation:

x	$f(x)$
Graph:	
Equation:	
Table:	
Domain:	Range:

3	$f(3)$
Graph:	
Equation:	
Table:	

Lesson Summary

In this lesson we wrote the equation of two functions given graphically and with a story context. We connected the domain and range to the context and the graph. We deepened our understanding of function notation, learning to interpret the notation for graphs, tables, and equations. We learned that functions can be added together both graphically and algebraically.

Unit 3, Lesson 5

Give Me Five

Learning Focus

Interpret function notation to match a function with its features.

How can I visualize a graph from knowing its features?

How can function notation describe a function?

How does function notation describe relationships between two functions?

Open Up the Math: Launch, Explore, Discuss

1. You have two sets of cards, A cards and B cards.

    a. Lay out all the A cards. Each A card gives you a representation of a function.

    b. Select a B card and read each statement aloud.

    c. Find an A card that matches all 5 statements on the B card and justify each statement.

    d. Record the match and your justification for it.

    e. Keep the pairs together for the next part of the lesson.

    Proceed until all the A and B cards are matched. Be sure that you can justify all 5 statements on the B card with the function that you match it with.

A card	B card	Reason
A1 matches with:		
A2 matches with:		
A3 matches with:		
A4 matches with:		
A5 matches with:		
A6 matches with:		

After you match the cards, complete the information below. Use both the A and the B cards in the pair for reference:

2. Complete this information about the pair of cards that includes A1

    Write the equation of $f(x)$ and use it to:

    Find an exact value for $f(3)$:

    Find where $f(x) = -1$:

3. Complete this information about the pair of cards that includes A2

    What are the intervals of increase and decrease for $f(x)$?

Increasing:	Decreasing:

4. Complete this information about the pair of cards that includes A3

   Graph $f(x) + g(x)$ from $[-1,3]$:

5. Complete this information about the pair of cards that includes A4

   Write the equation and graph $f(x)$:

6. Complete this information about the pair of cards that includes A5

   Which function is changing faster in the interval $[-5,1]$? Why?

7. Complete this information about the pair of cards that includes A6

   Write the equation for $g(x) + f(x)$:

   Find $g(3) + f(3)$:

Ready for More?

1. Explain the meaning of $f(a) = b$ and $g(a) = c$.
2. Find the value of $f(a) + g(a)$.

Takeaways

Common Phrases About Functions

We write:	We mean:

Lesson Summary

In this lesson we interpreted function notation to match features with functions. We learned common phrases that can be used to "translate" function notation depending on the context. For instance, $f(3) = 7$ can be interpreted as the height of the graph at $x = 3$ is 7 or that when $x = 3$ is substituted into $f(x)$, the output is 7.

Unit 3, Lesson 6

Another Day

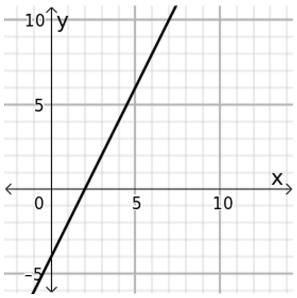
Jump Start

Which One Doesn't Belong? Examine each function and determine which one is not like the others. Be prepared to give a mathematical reason for your choice.

a.

x	$f(x)$
-2	1
-1	-1
0	-3
1	-5
2	-7
3	-9

b.



c.

x	$f(x)$
-2	-3
-1	-1
0	1
1	3
2	5
3	7

d. $g(x) = 2x$

Reason:

Learning Focus

Write and graph equations of functions.

Compare the graphs of related functions.

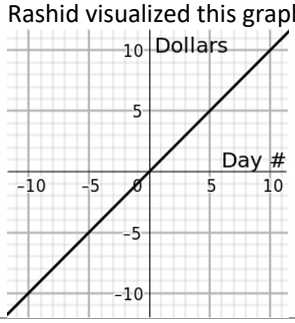
What is the effect on the graph when a number is added to an equation of a function?

Open Up the Math: Launch, Explore, Discuss

Every day when Rashid's mother comes home from work, he asks her how her day was and she says, "Another day, another dollar." Being a mathematical thinker, Rashid started imagining the function that would model his mother's saying. His thinking went something like this:

"On day 0, she hasn't worked, so she has 0.
On day 1, she works and gets \$1.
On day 2, she gets another dollar, so she has \$2.
One day 3, she gets another dollar, so she has \$3."

Rashid visualized this graph of the function:



1. Model the function that describes this pattern with a table and explicit equation.

As Rashid pursued this thinking, he wondered what would happen to his function if he started with $\$5$ on day 0 and then earned a dollar a day.

2. Model this situation with a table, graph, and explicit equation.

3. Compare the graph in problem 2 to Rashid's original graph. What do you notice?

Rashid thought about all the bills that his family needed to pay and wondered what the function would be if the situation changed so that on day 0, they owed $\$10$ and then added a dollar a day.

4. Write the explicit equation and graph the function that models this situation.

5. Compare the three graphs and equations. What do you notice?

Rashid thought, "A dollar a day is no way to get ahead. What if I start with $\$1$ on day 0 and double my dollars each day? That means I'd have $\$2$ on day 1, $\$4$ on day 2, and $\$8$ on day 3. I'd come home from work and say, "Another day, another double."

6. Let $D(t)$ be the function that models this situation. Represent $D(t)$ with a table, graph, and equation.

7. Rashid wondered what would happen to this function if he added $\$3$ to every output. What's your prediction?

8. Try it! Use an equation, a table, and a graph to model the function that adds 3 to every output of $D(t)$.

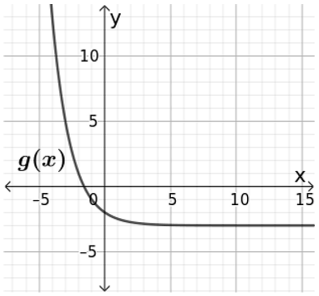
9. How does the graph of $D(t)$ compare to the new function, $D(t) + 3$?

10. If $f(x) = x$, what do you know about the graph of $f(x) + b$? (b represents any constant number).

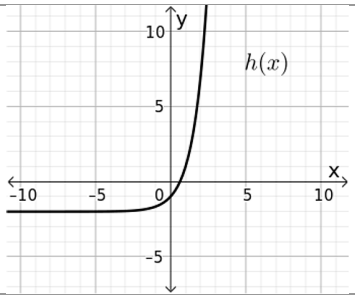
11. If $D(t) = 2^t$ what do you know about the graph of $D(t) + b$? (b represents any constant number).

12. Compare the features of $f(x)$ and $g(x)$.

$$f(x) = \left(\frac{1}{2}\right)^x + 1$$



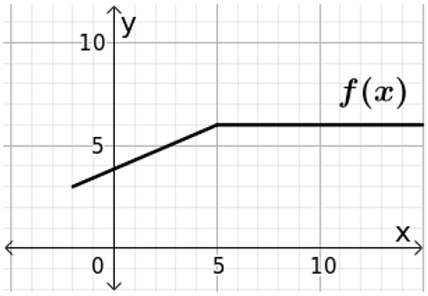
13. Compare the features of $h(x)$ and $m(x)$.



x	$m(x)$
-2	$\frac{1}{9} + 1 = \frac{10}{9}$
-1	$\frac{1}{3} + 1 = \frac{4}{3}$
0	$1 + 1 = 2$
1	$3 + 1 = 4$
2	$9 + 1 = 10$
3	$27 + 1 = 28$

Ready for More?

The graph of $f(x)$ is shown below. Sketch the graph of $g(x) = f(x) - 3$



Takeaways

Effects of a vertical translation on the linear parent function, $f(x) = x$:

- _____
- _____
- _____
- _____

Effects of a vertical translation on an exponential parent function, $f(x) = 2^x$:

- _____
- _____
- _____
- _____

Adding Notation, Vocabulary, and Conventions

Parent function: _____

Vertical translation: _____

Vocabulary

- **parent function**
- **transformations on a function (rigid)**
- **vertical shift**

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we learned that adding or subtracting a number to a function results in a vertical translation of the graph. In a vertical translation, functions do not change their shape, they are simply shifted up or down.