Unit 2, Lesson 1

Piggies and Pools

Learning Focus

Represent situations with different types of growth.

Compare models for situations that occur over time.

What type of situation can be modeled by a continuous graph? When is a graph of only separate points appropriate?

What are the similarities and differences between an arithmetic sequence and a linear function?

Can a geometric sequence be continuous?

Open Up the Math: Launch, Explore, Discuss

1. My little sister, Savannah, is three years old. She has a piggy bank that she wants to fill. She started with 5 pennies, and each day when I come home from school, she is excited when I give her 3 pennies that are left over from my lunch money. Use a table, a graph, and an equation to create a mathematical model for the number of pennies in the piggy bank on day n.

2. Our family has a small pool for relaxing in the summer that holds $1{,}500$ gallons of water. I decided to fill the pool for the summer. I was getting bored just standing there watching water flow and began to think about a mathematical model for the time it takes to fill the pool while I was waiting. I checked the flow on the hose and found that it was filling the pool at a rate of 2 gallons every minute. When I had 5 gallons of water in the pool, I started the timer. Use a table, a graph, and an equation to create a mathematical model for the number of gallons of water in the pool at t minutes.

3. I'm more sophisticated than my little sister, so I save my money in a bank account that pays me 3% interest on the money in the account at the end of each month. (If I take my money out before the end of the month, I don't earn any interest for the month.) I started the account with $\$50$ that I got for my birthday. Use a table, a graph, and an equation to create a mathematical model of the amount of money I will have in the account after m months.

4. At the end of the summer, I decided to drain the $1{,}500$-gallon swimming pool. I noticed that it drains faster when there is more water in the pool. That was interesting to me, so I decided to measure the rate at which it drains. I found that 3% was draining out of the pool every minute. Use a table, a graph, and an equation to create a mathematical model of the gallons of water in the pool at t minutes.

5. Compare problems 1 and 3. What similarities do you see? What differences do you notice?

   Similarities:

   Differences:

6. Compare problems 1 and 2. What similarities do you see? What differences do you notice?

   Similarities:

   Differences:

7. Compare problems 3 and 4. What similarities do you see? What differences do you notice?

   Similarities:

   Differences:

Ready for More?

Use your model in problem 4 to find when the pool will be empty. Justify your answer.

Takeaways

A geometric sequence _____

Exponential functions _____

Discrete functions _____

Continuous functions _____

Arithmetic and geometric sequences are _____

Adding Notation, Vocabulary, and Conventions

Domain of a function: _____

Vocabulary

- **continuous function/discontinuous function**
- **discrete function**
- **domain**
- **exponential function**
- **linear function**

Lesson Summary

In this lesson, we learned that the possible inputs for a function are called the domain. We found that some situations are best described using a discrete model and others are represented better with a continuous model. Arithmetic sequences are part of the linear family of functions and geometric sequences are part of the exponential family of functions.

Unit 2, Lesson 2

Shh! Please Be Discreet (Discrete)!

Jump Start

Which One Doesn't Belong? Determine which of the numbers below is not like the others and be prepared with a reason for your choice.

A. 2

B. -2

C. $-\frac{1}{2}$

D. $\sqrt{2}$

E. $2.\overline{22}$

Reason:

Learning Focus

Use representations to model situations with linear and exponential functions.

Determine when a discrete model or continuous model is most appropriate.

What is the difference between discrete and continuous functions?

How can I tell if a discrete or continuous model is best for a given situation?

Are all linear functions continuous? Are all arithmetic sequences discrete?

Are all exponential functions continuous? Are all geometric sequences discrete?

How is the domain related to whether the function is continuous or discrete?

Open Up the Math: Launch, Explore, Discuss

1. The Library of Congress in Washington D.C. is considered the largest library in the world. They often receive boxes of books to be added to their collection. Since books can be quite heavy, they aren't shipped in big boxes. On average, each box contains 8 books.

   If, on average, each box contains 8 books, how many books are received by the library in 6 boxes, 10 boxes, or n boxes?

   Use a table, a graph, and an equation to model this situation.

2. Many of the books at the Library of Congress are electronic. About 13 e-books can be downloaded onto the computer each hour.

   If about 13 e-books can be downloaded onto the computer each hour, how many e-books can be added to the library in 3 hours, 5 hours, or n hours (assuming the computer memory is not limited)?

   Use a table, a graph, and an equation to model this situation.

3. The librarians work to keep the library orderly and put books back into their proper places after they have been used. A librarian can sort and shelve 3 books in a minute.

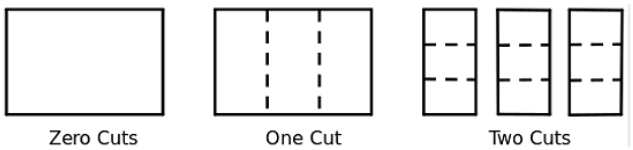
   If a librarian can sort and shelve 3 books in a minute, how many books does that librarian take care of in 3 hours, 5 hours, or n hours?

   Use a table, a graph, and an equation to model this situation.

4. Would it make sense in any of these situations for there to be a time when 32.5 books had been shipped, downloaded into the computer, or placed on the shelf? Explain.

5. Which of these situations (in problems 1–3) represent a discrete function and which represent a continuous function? Justify your answer.

6. A giant piece of paper is cut into 3 equal pieces, and then each of those is cut into 3 equal pieces and so forth.



   a. Use a table, a graph, and an equation to model this situation.

      How many papers will there be after a round of 10 cuts? 20 cuts? n cuts?

   b. Identify the domain of the function.

   c. Would it make sense to look for the number of pieces of paper at 5.2 cuts? Why?

   d. Would it make sense to look for the number of cuts it takes to make 53.6 papers? Why?

7. Medicine taken by a patient breaks down in the patient's blood stream and dissipates out of the patient's system. Suppose a dose of 60 milligrams of anti-parasite medicine is given to a dog and the medicine breaks down so that 20% of the medicine becomes ineffective every hour.

   a. Use a table, a graph, and an equation to model this situation.

      How much of the 60 milligram dose is still active in the dog's bloodstream after 3 hours, after 4.25 hours, and after n hours?

   b. Identify the domain of the function.

   c. Would it make sense to look for an amount of active medicine at 3.8 hours? Why?

   d. Would it make sense to look for when there is 35 milligrams of medicine? Why?

8. Which of the functions modeled in problems 6 and 7 are discrete and which are continuous? Why?

9. What needs to be considered when looking at a situation or context and deciding if it fits best with a discrete or continuous model?

10. Describe the differences in each representation (table, graph, and equation) for discrete and continuous functions.

11. Which of the functions modeled in this task are linear? Which are exponential? Why?

Ready for More?

On a separate piece of paper or a pair of index cards, work with your partner to create one continuous linear context and one discrete linear context. Include enough information that the context could be modeled with all the representations. When you finish, trade your cards with another pair of students for feedback. Discuss the following:

- Which context is discrete and which context is continuous?
- Is the context actually linear?

Takeaways

	Discrete	Continuous
Context Features		
Tables		
Graphs		
Equations		
Domain		

Adding Notation, Vocabulary, and Conventions

Set Builder Notation:

We write: _____

We say: _____

We mean: _____

Vocabulary

- **set builder notation**

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we modeled linear and exponential functions and learned to identify features that allow us to determine whether a discrete or a continuous model is more appropriate. We discussed number sets and used them to write function domains using set builder notation.

Unit 2, Lesson 3

The Name Game

Learning Focus

Use representations to determine if a function is discrete or continuous.

Determine the domain of a function.

How can I tell if a function is linear or exponential, given any representation?

How can I think about a constant rate of change or a constant ratio if a function is continuous?

Why are arithmetic sequences linear functions?

Why are geometric sequences exponential functions?

Open Up the Math: Launch, Explore, Discuss

In today's lesson, you're going to play the name game. You and your partner have been given a game board and a set of cards. Each large square on your game board has four smaller squares. Information about a function is given in the first square. In one of the squares, you are asked to produce some kind of information about the function. The last two squares will come from finding cards in your deck to match the function. By the time you finish a set, you will have filled in all four squares and each large square will have the following information:

1. The given function
2. The domain of the function and whether the function is discrete or continuous
3. Another representation of the function
4. The type of the function and the type of growth

To identify the type of function, you will need to understand the defining characteristics of linear and exponential functions:

a. Linear functions grow by equal differences over equal intervals.

b. Exponential functions grow by equal factors over equal intervals.

1. As you discuss the game with the class, record the results in the table below so that you have a record of your work.

Function	Function Characteristics
The given function: $y = -\frac{1}{4}x + 3$	The domain of the function and whether the function is discrete or continuous:
Another representation of the function:	Type of function and the difference or growth factor:

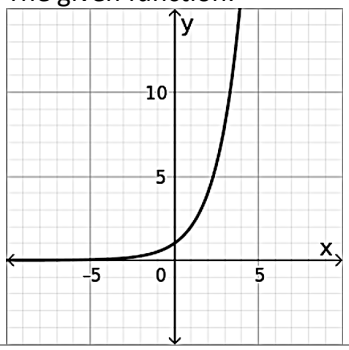
2. Complete:

Function	Function Characteristics
The given function: $f(1) = 1, f(n) = \frac{2}{3}f(n-1)$	The domain of the function and whether the function is discrete or continuous:
Another representation of the function:	Type of function and the difference or growth factor:

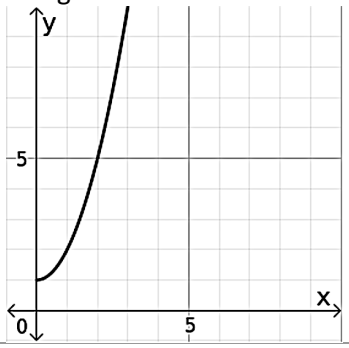
3. Complete:

Function	Function Characteristics
The given function: <table><tr><th>x</th><th>$f(x)$</th></tr><tr><td>-130</td><td>57.2</td></tr><tr><td>-124</td><td>54.56</td></tr><tr><td>-120</td><td>52.8</td></tr><tr><td>-114</td><td>50.16</td></tr><tr><td>-110</td><td>48.4</td></tr><tr><td>-107</td><td>47.08</td></tr><tr><td>-100</td><td>44</td></tr></table>	The domain of the function and whether the function is discrete or continuous.
Another representation of the function:	Type of function and the difference or growth factor:

4. Complete:

Function	Function Characteristics
The given function: 	The domain of the function and whether the function is discrete or continuous:
Another representation of the function:	Type of function and the difference or growth factor:

5. Complete:

Function	Function Characteristics
The given function: 	The domain of the function and whether the function is discrete or continuous:
Another representation of the function:	Type of function and the difference or growth factor:

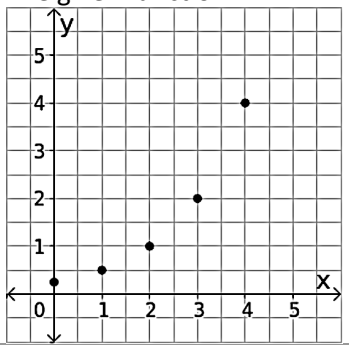
6. Complete:

Function	Function Characteristics
The given function: A bubble gum machine in a grocery store is filled with 500 candies. Each time a person puts in money, 10 candies come out. This function is a model for the number of candies left in the machine after a person buys some candy.	The domain of the function and whether the function is discrete or continuous:
Another representation of the function:	Type of function and the difference or growth factor:

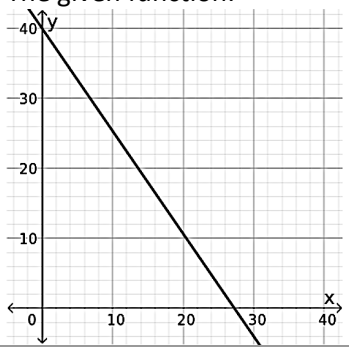
7. Complete:

Function	Function Characteristics
The given function: A radioactive element decays at a rate of 5% each year. The scientists begin with 40 grams of the element. The function is a model for the amount of the element remaining at a given time.	The domain of the function and whether the function is discrete or continuous:
Another representation of the function:	Type of function and the difference or growth factor:

8. Complete:

Function	Function Characteristics
The given function: 	The domain of the function and whether the function is discrete or continuous:
Another representation of the function:	Type of function and the difference or growth factor:

9. Complete:

Function	Function Characteristics
The given function: 	The domain of the function and whether the function is discrete or continuous:
Another representation of the function:	Type of function and the difference or growth factor:

Ready for More?

Make an additional representation for each function given in problems 1–9.

Takeaways

Identifying linear and exponential functions:

Lesson Summary

In this lesson, we used the definitions of linear and exponential functions to determine if functions were linear or exponential. We learned to identify equal intervals on a continuous function so that we could tell if there were equal differences or change factors. We practiced determining domains and whether a function is discrete or continuous.

Unit 2, Lesson 4

The In-Betweeners

Jump Start

1. Find the missing terms in this geometric sequence:

x	1	2	3	4
y	5			320

2. Write an explicit equation for the geometric sequence given in the table from problem 1.

3. Complete:

   I know $2^3 \cdot 2^5 =$ _____, because _____.

4. In general, $2^m \cdot 2^n =$

Learning Focus

Explore the meaning of a fraction as an exponent.

How can you determine the values of an exponential function that occur between whole number inputs?

Since the domain of a continuous exponential function includes all rational numbers, how do we interpret a fraction as an exponent?

Open Up the Math: Launch, Explore, Discuss

In previous lessons, you've worked with contexts for continuous exponential functions, such as the decay of medication in the bloodstream or earning interest over time in a bank account. Since time is continuous, we need to start thinking about the numbers that fill in between the values like 2^2 and 2^3 in an exponential function. These numbers are actually quite interesting, so we're going to do some exploring in this task to see what we can find out about these "in-betweeners."

Let's begin in a familiar place:

1. Complete the following table for a geometric sequence defined recursively as: $f(0) = 4$ and $f(n) = 2 \cdot f(n-1)$

x	0	1	2	3	4
$f(x)$	4				

2. Write an explicit equation for the function defined by the table.

3. Plot these points on the graph at the end of this task and sketch the graph of $f(x)$.

We know a geometric sequence is discrete, with a domain consisting only of the whole numbers. Travis and Tehani, two inquisitive math students, are wondering what would happen if they decided they wanted this table to be just a few points on a continuous, exponential function. Then they could create a table with more entries, maybe with a point halfway between each of the points in the table above. There are a couple of ways that they might think about filling in the half steps.

Travis makes the following claim:

"If the function doubles each time x goes up by 1, then half that growth occurs between 0 and $\frac{1}{2}$ and the other half occurs between $\frac{1}{2}$ and 1.

"So for example, we can find the output at $x = \frac{1}{2}$ by finding the average of the outputs at $x = 0$ and $x = 1$."

4. Fill in the parts of the table below that you've already computed, and then decide how you might use Travis's strategy to fill in the missing data. Also plot Travis's data on the graph at the end of the task (problem 12).

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
$f(x)$	4								

5. Comment on Travis's idea. How does it compare to the table generated in problem 1? For what kind of function would this reasoning work?

Tehani suggests they should fill in the data in the table in the following way:

"I noticed that the function increases by the same factor each time x goes up 1, and I think this property should hold over each half-interval as well."

6. Fill in the parts of the table below that you've already computed in problem 1, and then decide how you might use Tehani's new strategy to fill in the missing data. As in the table in problem 1, each entry should be multiplied by some constant factor to get the next entry, and that factor should produce the same results as those already recorded in the table. Use this constant factor to complete the table. Also plot Tehani's data on the graph at the end of this task (problem 12).

   Tehani's table:

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
$f(x)$	4								

7. What if Tehani wanted to find values for the function every third of the interval instead of every half? What constant factor would she use for every third of an interval to be consistent with the function doubling as x increases by 1? Use this multiplier to complete the following table.

x	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2	$\frac{7}{3}$	$\frac{8}{3}$	3
$f(x)$										

8. What number did you use as a constant factor to complete the table in problem 6?

9. What number did you use as a constant factor to complete the table in problem 7?

Pause and Reflect

10. Tehani and Travis know that the values in her tables should fit the function rule written in problem 2. For example:

    a. Complete:

    $f\left(\frac{1}{2}\right) = 4 \cdot 2^{\frac{1}{2}} =$ _ and $f\left(\frac{3}{2}\right) = 4 \cdot 2^{\frac{3}{2}} =$ _

    b. Complete:

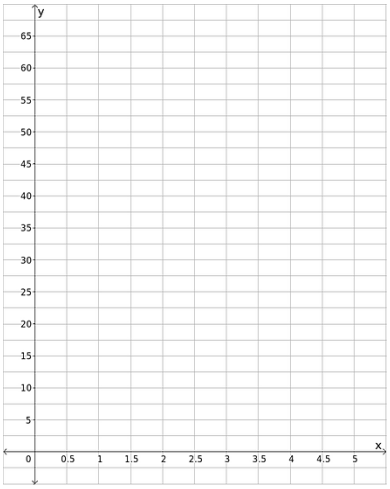
    $f\left(\frac{1}{3}\right) = 4 \cdot 2^{\frac{1}{3}} =$ _ and $f\left(\frac{2}{3}\right) = 4 \cdot 2^{\frac{2}{3}} =$ _

    c. What does this suggest about the value of $2^{\frac{1}{2}}$?

    d. What does this suggest about the value of $2^{\frac{1}{3}}$?

11. Give a detailed description of how you would estimate the output value $f(x)$, for $x = \frac{5}{3}$.

12. Use the graph to record information requested in problems 4 and 6.



Ready for More?

Consider your work with prior exponential relationships and do your best to connect a story with this expression: $50(1.03)^{\frac{1}{2}}$.

Takeaways

In the past, I have used positive integers as exponents. For example: _____

Today, I found a meaning for rational exponents that are non-integer fractions, as in this example:

_____

I can justify that $2^{\frac{1}{2}} = \sqrt{2}$ as follows:

In general, our new property of exponents can be summarized as: _____

Vocabulary

- cube root
- number sets (systems)
- **rational exponent (fractional exponent)**
- square root

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we developed meaning for using a fraction as an exponent. We began with a geometric sequence and found a way to fit data at the points halfway between whole number inputs in a way that would maintain the multiplicative behavior of the sequence. We continued to work with different fractional inputs and gave meaning to using a variety of fractions as exponents.

Unit 2, Lesson 5

Twin Powers

Jump Start

Illustrate with an example why each of the following properties of exponents works:

1. $a^m \cdot a^n = a^{m+n}$
2. $(a^m)^n = a^{mn}$
3. $\frac{a^m}{a^n} = a^{m-n}$
4. $(a \cdot b)^m = a^m \cdot b^m$
5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Apply rule 3 to complete these statements:

6. Complete:

   By rule 3, $\frac{a^3}{a^3} =$ _ . By dividing out common factors in fractions, $\frac{a^3}{a^3} =$ _ .

   Therefore, _ .

7. Complete:

   By rule 3, $\frac{a^3}{a^4} =$ _ . By dividing out common factors in fractions, $\frac{a^3}{a^4} =$ _ .

   Therefore, _ .

Learning Focus

Relate the key features of exponential functions to properties of negative exponents.

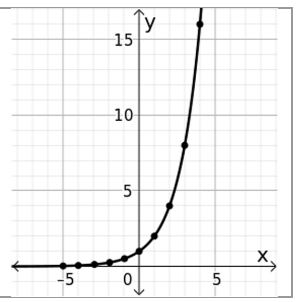
Rewrite exponential expressions that involve negative exponents.

How does my understanding of the properties of exponents help me explain the key features of exponential functions?

How does my understanding of the properties of exponents help me rewrite exponential expressions that contain negative exponents?

Open Up the Math: Launch, Explore, Discuss

Carlos and Clarita have been working on their math together and noticed something interesting.
Carlos: I've been looking at the graph of $f(x) = 2^x$ on my calculator and trying to make sense of the graph when the inputs are negative. Here's what I see on the graph:



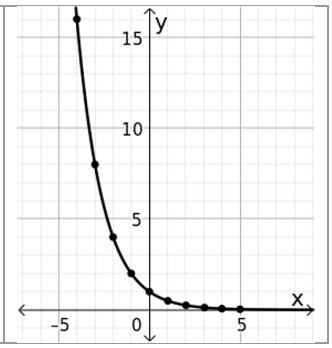
1. What observations do you make about Carlos's graph, especially when the x-values are negative?

Clarita: Sometimes it helps to make a table and look for patterns in the numbers. I'm going to try that.

2. Complete Clarita's table, making sure that your table matches the graph. Record any patterns that you see. **Use fractions instead of decimals throughout the task to make the patterns easier to see.**

x	4	3	2	1	0	-1	-2	-3	-4
$f(x) = 2^x$	16	8	4	2	1				

Carlos: So, here's what I think is really strange. Look at the graph of $g(x) = \left(\frac{1}{2}\right)^x$.



3. Compare the graphs of $g(x) = \left(\frac{1}{2}\right)^x$ and $f(x) = 2^x$. What similarities and differences do you notice?

Clarita: I think we should make a table to compare the two functions. I've added a column to my previous table.

4.

   a. Complete Clarita's table and make sure that the table matches the graphs of the functions.

x	4	3	2	1	0	-1	-2	-3	-4
$f(x) = 2^x$	16	8	4	2	1				
$g(x) = (\frac{1}{2})^x$									

   b. What similarities and differences do you notice in the table values?

      How would you explain the similarities and differences that you have observed?

Carlos: I remember this about negative exponents: $a^{-n} = \frac{1}{a^n}$. Negative exponents always have a "twin": an equivalent expression that is written with a positive exponent.

5. Explain how what Carlos remembers about negative exponents relates to the relationships you have observed in $g(x) = \left(\frac{1}{2}\right)^x$ and $f(x) = 2^x$ or to the work with negative exponents in the previous task.

Clarita: I think we could play around with these negative exponents and use them with bases other than 2.

6. Try applying the multiplication rule for exponents, $a^m \cdot a^n = a^{m+n}$, to these expressions. When you have finished, use another method to verify that your answers are correct.

   a. $2^5 \cdot 2^{-3}$

b. $3^2 \cdot 3^{-2}$

7. Try applying the division rule for exponents, $\frac{a^m}{a^n} = a^{m-n}$, to these expressions. When you have finished, use another method to verify that your answers are correct.

   a. $\frac{3^2}{3^{-1}}$

   b. $\frac{5^{-2}}{5^3}$

8. Try applying the power of a power rule, $(a^m)^n = a^{mn}$, for exponents to these expressions. When you have finished, use another method to verify that your answers are correct.

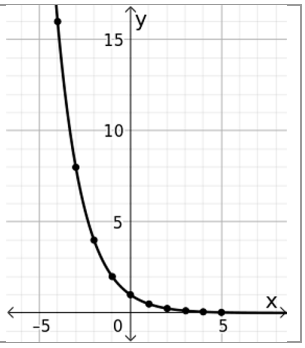
   a. $(3^3)^{-2}$

   b. $(5^{-1})^{-2}$

Carlos: All of this work with negative exponents has made me realize something—I can write two different exponential rules for the same function!

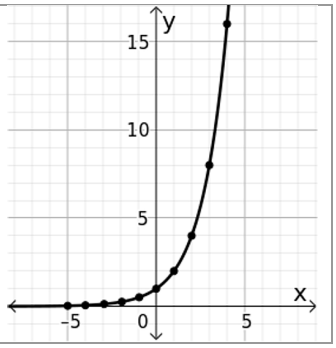
9.

Carlos has already written the rule $g(x) = \frac{1}{2}^x$ for this function. What is the other rule he is thinking about?



10.

Can you also write two different rules for this function?



Clarita: Wow, that's interesting. So how can I tell if an exponential function is always increasing or always decreasing before I graph it?

11. How would you answer Clarita's question?

Ready for More?

Let's think about negative exponents with negative bases. Find values for the following:

1. $(-3)^{-2}$
2. $(-3)^{-3}$
3. -3^{-2}
4. What would happen if we try to use -3 as the base of an exponential function? Plot a few points on the graph of $f(x) = (-3)^x$. Be sure to try positive and negative integers, as well as fractions, for x.

Takeaways

I noticed some useful strategies for working with expressions that include negative exponents, such as:

I noticed several things about exponential functions over the past few days, and each of these observations can be supported by the properties of exponents:

The end behavior on one end of an exponential function approaches the x-axis as an asymptote:

Vocabulary

- **asymptote**
- **horizontal asymptote**

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we noticed several characteristics of the graphs and tables of exponential functions that can be explained using our understanding of negative exponents. We also used the rules of exponents to change the form of numeric expressions that contain negative exponents.

Unit 2, Lesson 6

Half Interested or More Interesting

Jump Start

Notice and Wonder

Previously, you examined the following context:

Medicine taken by a patient breaks down in the patient's blood stream and dissipates out of the patient's system. Suppose a dose of 60 milligrams of anti-parasite medicine is given to a dog and the medicine breaks down such that 20% of the medicine becomes ineffective every hour. How much of the 60 milligram dose is still active in the dog's bloodstream after 3 hours, after 4.25 hours, and after n hours?

Here are three representations of that context. List at least two things that you notice and one thing you are wondering about relative to these representations.

Representation #1:

Hours	Amount	
0	60	$\times 0.8$
1	48	$\times 0.8$
2	38.4	$\times 0.8$
3	30.72	$\times 0.8$
4	24.58	$\times 0.8$
5	19.66	

Representation #2:

Hours	Amount	
0	60	$60 - 0.2(60)$
1	48	$48 - 0.2(48)$
2	38.4	$38.4 - 0.2(38.4)$
3	30.72	$30.72 - 0.2(30.72)$
4	24.58	$24.58 - 0.2(24.58)$
5	19.66	

Representation #3:

$A(t) = 60(0.8)^t$

Learning Focus

Examine how the properties of exponents work with rational exponents.

Write equivalent exponential functions using different growth factors.

What do rational exponents and negative exponents mean in contexts?

Do the laws of exponents work with rational exponents?

How does the growth factor change if we focus on a month of exponential growth instead of a year?

Open Up the Math: Launch, Explore, Discuss

Carlos and Clarita, the Martinez twins, have run a summer business every year for the past 5 years. Their first business, a neighborhood lemonade stand, earned a small profit that their father insisted they deposit in a savings account at the local bank. When the Martinez family moved a few months later, the twins decided to leave the money in the bank where it has been earning 5% interest annually. Carlos was reminded of the money when he found the annual bank statement they had received in the mail.

"Remember how Dad said we could withdraw this money from the bank when we are 20 years old," Carlos said to Clarita. "We have $\$382.88$ in the account now. I wonder how much that will be 5 years from now?"

1. Carlos calculates the value of the account 1 year at a time. He has just finished calculating the value of the account for the first 4 years. Describe how he can find the next year's balance and record that value in the table.

Year	Amount
0	382.88
1	402.02
2	422.12
3	443.23
4	465.39
5	

2. Clarita thinks Carlos is silly calculating the value of the account one year at a time, and says that he could have written a formula for the nth year and then evaluated his formula when $n = 5$. Write Clarita's formula for the nth year and use it to find the account balance at the end of year 5.

3. Carlos was surprised that Clarita's formula gave the same account balance as his year-by-year strategy. Explain, in a way that would convince Carlos, why this is so.

"I can't remember how much money we earned that summer," said Carlos. "I wonder if we can figure out how much we deposited in the account five years ago, knowing the account balance now?"

4. Carlos continued to use his strategy to extend his table year-by-year back 5 years. Explain what you think Carlos is doing to find his table values one year at a time and continue filling in the table until you get to -5, which Carlos uses to represent "5 years ago."

Year	Amount
-5	
-4	
-3	
-2	
-1	364.65
0	382.88
1	402.02
2	422.12
3	443.23
4	465.39
5	

Explanation:

5. Clarita evaluated her formula for $n = -5$. Again, Carlos is surprised that they get the same results. Explain why Clarita's method works.

Clarita doesn't think leaving the money in the bank for another 5 years is such a great idea and suggests that they invest the money in their next summer business. "We'll have some start-up costs, and this will pay for them without having to withdraw money from our other accounts."

Carlos remarked, "But we'll be withdrawing our money halfway through the year. Do you think we'll lose out on this year's interest?"

"No, they'll pay us a half-year portion of our interest," replied Clarita.

"But how much will that be?" asked Carlos.

6. Calculate the account balance and how much interest you think Carlos and Clarita should be paid if they withdraw their money $\frac{1}{2}$ year from now. Remember that they currently have $\$382.88$ in the account and that they earn 5% annually. Describe your strategy.

Clarita used this strategy: She substituted $\frac{1}{2}$ for n in the formula $A = 382.88(1.05)^n$ and recorded this as the account balance.

Carlos had some questions about Clarita's strategy:

- What numerical amount do we multiply by when we use $(1.05)^{\frac{1}{2}}$ as a factor?
- What happens if we multiply by $(1.05)^{\frac{1}{2}}$ and then multiply the result by $(1.05)^{\frac{1}{2}}$ again? Shouldn't that be a full year's worth of interest? Is it?
- If multiplying by $(1.05)^{\frac{1}{2}} \cdot (1.05)^{\frac{1}{2}}$ is the same as multiplying by 1.05, what does that suggest about the value of $(1.05)^{\frac{1}{2}}$?

7. Answer each of Carlos's questions listed as best as you can.

Pause and Reflect

As Carlos is reflecting on this work, Clarita notices the date on the bank statement that started this whole conversation. "This bank statement is three months old!" she exclaims. "That means the bank will owe us $\frac{3}{4}$ of a year's interest."

"So how much interest will the bank owe us then?" asked Carlos.

8. Find as many ways as you can to answer Carlos's question: How much will their account be worth in $\frac{3}{4}$ of a year (nine months) if it earns 5% annually and is currently worth $\$382.88$?

Carlos now knows he can calculate the amount of interest earned on an account in smaller increments than one full year. He would like to determine how much money is in an account each month that earns 5% annually with an initial deposit of $\$300$.

He starts by considering the amount in the account each month during the first year. He knows that by the end of the year the account balance should be $\$315$, since it increases 5% during the year.

9. Complete the table showing what amount is in the account each month during the first 12 months.

Deposit	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{5}{12}$	$\frac{6}{12}$	$\frac{7}{12}$	$\frac{8}{12}$	$\frac{9}{12}$	$\frac{10}{12}$	$\frac{11}{12}$	1 year
$\$300$												$\$315$

10. What number did you multiply the account by each month to get the next month's balance?

Carlos knows the exponential equation that gives the account balance for this account on an annual basis is $A = 300(1.05)^t$. Based on his work finding the account balance each month, Carlos writes the following equation for the same account: $A = 300\left(1.05^{\frac{1}{12}}\right)^{12t}$.

11. Verify that both equations give the same results. Using the properties of exponents, explain why these two equations are equivalent.

12. What is the meaning of the $12t$ in this equation?

Carlos shows his equation to Clarita. She suggests his equation could also be approximated by $A = 300(1.004)^{12t}$, since $1.05^{\frac{1}{12}} = 1.004$. Carlos replies, "I know the 1.05 in the equation $A = 300(1.05)^t$ means I am earning 5% interest annually, but what does the 1.004 mean in your equation?"

13. Answer Carlos's question. What does the 1.004 mean in $A = 300(1.004)^{12t}$?

The properties of exponents can be used to explain why $[(1.05)^{\frac{1}{12}}]^{12t} = 1.05^t$. Here are some more examples of using the properties of exponents with rational exponents. For each of the following, rewrite the expression using the properties of exponents and explain what the expression means in terms of the context.

14. $(1.05)^{\frac{1}{12}} \cdot (1.05)^{\frac{1}{12}} \cdot (1.05)^{\frac{1}{12}}$

15. $[(1.05)^{\frac{1}{12}}]^6$

16. $(1.05)^{\frac{-1}{12}}$

17. $(1.05)^2 \cdot (1.05)^{\frac{1}{4}}$

18. $\frac{(1.05)^2}{(1.05)^{\frac{1}{2}}}$

Ready for More?

Use $[(1.05)^{\frac{1}{12}}]^{12} = 1.05$ to explain why $(1.05)^{\frac{1}{12}} = \sqrt[12]{1.05}$.

Takeaways

The following properties of exponents that make sense for positive integer exponents also apply and make sense for negative integer exponents and for fractional exponents:

We can interpret the negative exponential factor in ab^{-x} as meaning:

We can interpret a fractional exponential factor in $ab^{\frac{m}{n}}$ as meaning:

Lesson Summary

In this lesson, we continued to explore the meaning of rational exponents, including negative integer exponents and fractional exponents. We learned that the properties of exponents can be applied to all rational exponents, not just integer exponents.

Unit 2, Lesson 7
Radical Ideas

Jump Start

If n is a positive integer greater than 1 and both a and b are positive real numbers, then,

a. $\sqrt[n]{a^n} = a$

b. $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

c. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

1. Illustrate each of these rules for radicals with an example using numbers.

2. Show why each of these rules is true by rewriting the radical expressions using rational exponents.

Learning Focus

Change the form of radical expressions using properties of exponents.

Which is a more efficient way to change the form of a radical expression: using radicals or using exponents?

How do the properties of exponents help explain methods for representing and manipulating radicals?

Open Up the Math: Launch, Explore, Discuss

Now that Tia and Tehani know that $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$, they are wondering which form, radical form or exponential form, is best to use when working with numeric and algebraic radical expressions.

Tia says she prefers radicals since she understands the following properties for radicals (and there are not too many properties to remember):

If n is a positive integer greater than 1 and both a and b are positive real numbers, then,

- $\sqrt[n]{a^n} = a$
- $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
- $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
- Tehani says she prefers exponents since she understands the following properties for exponents (and there are more properties to work with):
- $a^m \cdot a^n = a^{m+n}$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$
- $a^{-n} = \frac{1}{a^n}$

Both Tia and Tehani agree that it is important to understand the following rule that expresses the relationship between radicals and exponents:

$a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$

Using their preferred notation, Tia might rewrite $\sqrt[3]{x^8}$ as follows:

$\sqrt[3]{x^8} = \sqrt[3]{x^3 \cdot x^3 \cdot x^2} = \sqrt[3]{x^3} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x^2} = x \cdot x \cdot \sqrt[3]{x^2} = x^2 \cdot \sqrt[3]{x^2}$

(Tehani points out that Tia also used some exponent rules in her work.)

On the other hand, Tehani might rewrite $\sqrt[3]{x^8}$ as follows:

$\sqrt[3]{x^8} = x^{\frac{8}{3}} = x^{2+\frac{2}{3}} = x^2 \cdot x^{\frac{2}{3}}$ or $x^2 \cdot \sqrt[3]{x^2}$

For each of the following problems, change the form of the expression in the ways you think Tia and Tehani might do it.

1. Original Expression:

   $\sqrt{27}$

   What Tia and Tehani might do to change the form of the expression:

   Tia's method:

   Tehani's method:

2. Original Expression:

   $\sqrt[3]{32}$

   What Tia and Tehani might do to change the form of the expression:

   Tia's method:

   Tehani's method:

3. Original Expression:

   $\sqrt{20x^7}$

   What Tia and Tehani might do to change the form of the expression:

   Tia's method:

   Tehani's method:

4. Original Expression:

   $\sqrt[3]{\frac{16xy^5}{x^7y^2}}$

   What Tia and Tehani might do to change the form of the expression:

   Tia's method:

   Tehani's method:

One of the conventions in the math world is to rewrite expressions so they do not have negative exponents. One of the easiest ways to get started is to rewrite the negative exponents as positive in the first step, but you can work with each expression in the way you choose. In each step, write a justification for the step. Here, an example of one is entirely worked out.

	Problem:	Step 1:	Step 2:	Answer:
EXAMPLE	$\frac{x}{x^{-3}}$	Rewrite with positive exponents. $x \cdot x^3$ Because $x^3 = \frac{1}{x^{-3}}$	Apply the exponent rules. x^{1+3} Exponent multiplication rule	x^4

5. Complete:

Problem:	Step 1:	Step 2:	Answer:
$\frac{y^{-3}}{y^{-1}}$	Rewrite with positive exponents.	Apply the exponent rules.	$\frac{1}{y^2}$

6. Complete:

Problem:	Step 1:	Step 2:	Answer:
$\frac{3x^{-9}}{6x^{-11}}$			$\frac{x^2}{2}$

7. Complete:

Problem:	Step 1:	Step 2:	Answer:
$\frac{a^2b^{-3}}{a^3b^{-1}}$			

8. Complete:

Problem:	Steps			Answer:
$\left(\frac{2x}{y}\right)^{-5}$	Raise everything to the -5 power.	Rewrite with positive exponents.	Combine common factors.	

9. Complete:

Problem:	Steps			Answer:
$\left(\frac{c^4d}{c^{-2}}\right)^{-4}$				

Ready for More?

Now we have a challenge: The answer to a problem is given in the last column. Your job is to create "the problem." **The problem must be equivalent to the answer given and contain negative exponents.** After writing the problem, show how to change the form to get the answer provided. Challenge yourself to come up with the trickiest problems that you can!

Problem:	Steps:			Answer:
				$\frac{3d^7}{25c^6}$
				$\frac{x^2z^7}{y^5}$
				1

Takeaways

Strategies for rewriting radical expressions, root n, in equivalent forms:

Strategies for changing the form of exponential expressions:

Vocabulary

- radical

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we learned how to change the form of complicated radical and exponential expressions using the properties of radicals and exponents. Strategies for changing the form of radical expressions can be explained by converting the radical expressions to exponential form.

Unit 2, Lesson 8

Getting Down to Business

Learning Focus

Make modeling decisions about business plans.

Interpret mathematical models to make business decisions.

Determine which type of function grows faster and make arguments about why.

Which type of function increases faster—linear or exponential?

Which model is best for a given situation, discrete or continuous?

How can mathematical models help to make business decisions?

Open Up the Math: Launch, Explore, Discuss

Calcu-rama had a net income of $\$5$ million in 2020, while a small competing company, Computafest, had a net income of $\$2$ million. The management of Calcu-rama develops a business plan for future growth that projects an increase in net income of $\$0.5$ million per year, while the management of Computafest develops a plan aimed at increasing its net income by 15% each year.

1. Create standard mathematical models (table, graph, and equations) for the projected net income over time for both companies.

2. Compare the two companies.

3. If both companies were able to meet their net income growth goals, which company would you choose to invest in? Why?

4. When, if ever, would your projections suggest that the two companies have the same net income? How did you find this? Will they ever have the same net income again?

5. Since we are creating the models for these companies, we can choose to have a discrete model or a continuous model. What are the advantages or disadvantages for each type of model?

6. Write the domain of each function, based on your model.

Ready for More?

In this task, we compared a linear and an exponential function and found that the exponential function was greater for large values of t.

Can you find a linear function that exceeds an increasing exponential function for very large values of t? Use technology to investigate and record your results here.

Takeaways

Linear Function

Definition:	**It's not a linear function if:**

Characteristics:

Equations: • Explicit: _____ • Recursive: _____	Graphs: _____ Tables: _____ Contexts: _____ Other: _____

Exponential Function

Definition:	**It's not an exponential function if:**

Characteristics:

Equations: • Explicit: _____ • Recursive: _____ Tables: _____ Contexts: _____ Other: _____	Graphs:

Lesson Summary

In this lesson, we modeled the growth of two businesses and made comparisons. We used our representation to find when the two businesses had the same net income and to justify which business was the best investment. We found that exponential functions exceed linear functions for large values of x and that the point of intersection of the two functions has meaning in a realistic context.

Unit 2, Lesson 9

I Can See—Can't You?

Learning Focus

Understand and find the average rate of change of a function in an interval.

Develop a formula for the average rate of change for any function.

How can I find the average rate of change of a function that is not linear?

What does the average rate of change mean?

Open Up the Math: Launch, Explore, Discuss

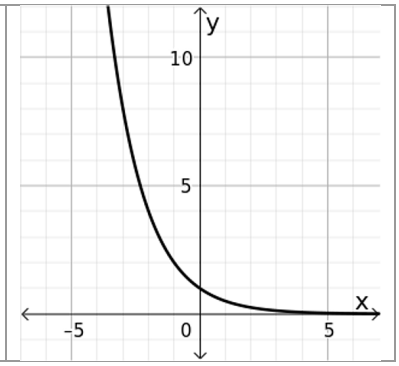
Kwan's parents bought a home for $50,000 in 2007 just as real estate values in the area started to rise quickly. Each year, their house was worth more until they sold the home in 2017 for $309,587.

1. Model the growth of the home's value from 2007 to 2017 with both a linear and an exponential equation. Then graph the two models on the graph.
2. What was the change in the home's value from 2007 to 2017?

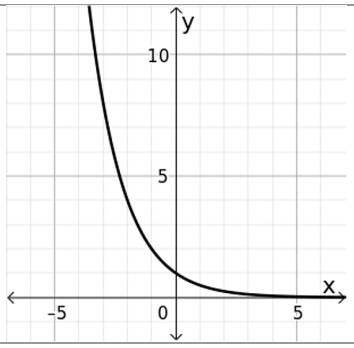
   The average rate of change is defined as the change in y (or $f(x)$) divided by the change in x.
3. What was the average rate of change of the linear function from 2007 to 2017?
4. What is the average rate of change of the exponential function in the interval from 2007 to 2017?
5. How do the average rates of change from 2007 to 2017 compare for the two functions? Explain.
6. What was the average rate of change of the linear function from 2007 to 2012?
7. What is the average rate of change of the exponential function in the interval from 2007 to 2012?
8. How do the average rates of change from 2007 to 2012 compare for the two functions? Explain.
9. How can you use the equation of the exponential function to find the average rate of change over a given interval?

   How does this process compare to finding the slope of the line through the endpoints of the interval?

Consider the graph:



10. What is the equation of the graph shown?
11. What is the average rate of change of this function on the interval from $x = -3$ to $x = 1$?
12. What is the average rate of change of this function in the interval from $x = -3$ to $x = 0$?
13. What is the average rate of change of this function in the interval from $x = -3$ to $x = -1$?
14. What is the average rate of change of this function in the interval from $x = -3$ to $x = -1.5$?
15. Draw the line through the point at the beginning and end of each of the intervals in 11, 12, 13, and 14. What is the slope of each of these lines?



16. Which of these average rates of change best represents the change at the point $(-2,4)$?

    Explain your answer.
17. How does the average rate of change compare to the change factor for an exponential function? What is described by each of these quantities?

Ready for More?

It makes sense to some people that the way to find an average rate of change of a function would be to:

- find the rate of change at the beginning of the interval,
- find the rate of change at the end of the interval,
- add the two rates together, and
- divide by 2.

Does this strategy ever work? Explain why or why not.

Takeaways

Average rate of change of a function over the interval $[a, b]$:

This formula means:

Secant line:

Vocabulary

- **average rate of change**
- **secant line**
- tangent to a curve

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we learned to find the average rate of change of a function in an interval. We learned that the average rate of change is calculated by finding the change in y and dividing by the change in x. Sometimes an equation is used to calculate the y-values at the beginning and end of the interval, and sometimes a graph is used to find the heights at the beginning and end of the interval. In either cases, the y-values are subtracted and then that amount is divided by the width of the interval, or the difference between the x-values.

Unit 2, Lesson 10

Point the Way

Learning Focus

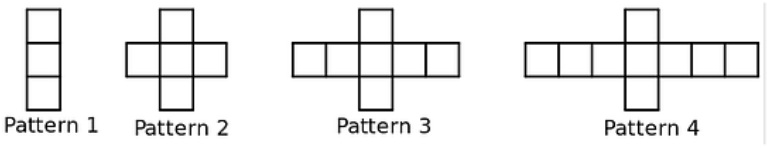
Find patterns that are useful in writing equations for linear functions.

Are there different ways to write the equation of a line?

What does each part of an equation tell us?

Open Up the Math: Launch, Explore, Discuss

Grandma Billings is making a quilt and piecing together a number of blocks. Zac and Sione are watching and thinking about predicting the number of quilt blocks in this pattern:



Zac said: I noticed that two blocks are being added each time. I thought it would be easier if I had the y-intercept, so I imagined what "Pattern 0" would look like by subtracting 2. That left 1 block at $x = 0$, so I wrote $B = 2n + 1$.

Sione said: I also noticed that the pattern was growing by two each time. I saw that Pattern 1 just has 3 blocks. Pattern 2 has 3 blocks plus 2 groups of 1 block on either side. Pattern 3 has 3 blocks plus 2 groups of 2 blocks. So, I wrote $B = 2(n - 1) + 3$.

1. What does $2(n - 1)$ mean in Sione's equation?

2. Are the equations that Zac and Sione wrote equivalent? How do you know?

Marcus joined the conversation and noticed this: I was focusing on the strip of blocks in the middle and looking at Patterns 3 and 4. I think that you could also write $B = 2(n - 2) + 5$, and $B = 2(n - 3) + 7$.

3. Do you agree with Marcus? Are these equations all equivalent to Zac's equation?

Sione said, "What is going on here? I'm going to look at a table to see if I can see why this is working." Sione made this table and noticed a pattern in the table and the equations they had written.

n	0	1	2	3	4
$f(n)$	1	$1 + 2$	$1 + 2 + 2 = 5$	$1 + 2 + 2 + 2 = 7$	$1 + 2 + 2 + 2 + 2 = 9$

Sione said, "I used the pattern I saw in the table to write another equation. This time, it's $f(n) = 2(n - 4) + 9$."

Zac said, "I don't even need the table. I think $f(n) = 2(n - 5) + 11$ is another equivalent equation."

4. Examine the table and compare it to the equations that have been written. Explain how Sione might have been using the table when he found the equation $y = 2(n - 4) + 9$.

5. Use Sione's method to find an equation for each of the tables below.

a.

x	y
32	50
33	55
34	60

b.

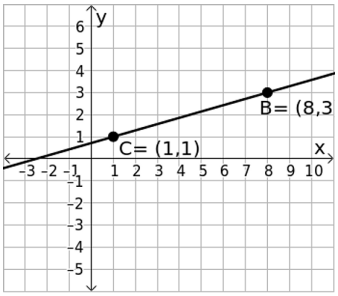
x	y
26	78
28	70
30	62

c.

x	y
-15	32
-14	39
-13	46

6. What information do you need to write an equation using this method?

7. Write the equation of the line shown in the graph using this method.



8. Use this method to write the equation of the line with slope -3 that contains the point $(-1, -5)$.

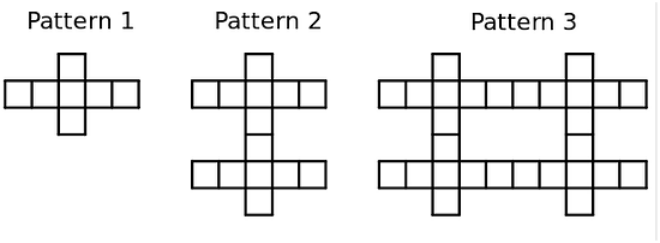
9. Zac has a challenge: "I'll bet you can't use the pattern to write the equation of the line through the points $(1, -3)$ and $(3, -5)$. Try it!"

10. Sione is still thinking about the pattern: "I wonder if we could use this pattern to graph lines, thinking of the starting point and using the slope." Try it with the equation: $f(x) = -2(x + 1) - 3$.

11. Zac wonders, "What is it about lines that makes all this work?" How would you answer Zac?

Ready for More?

Grandma Billings has started piecing her quilt together and has created the following growth pattern:



See if you can find at least two equivalent equations to model the number of squares in the pattern.

Takeaways

Point-slope form comes from the slope formula: _____

With a little rearranging:

Vocabulary

- **point-slope form of a line**

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we learned a new and efficient pattern for writing the equation of a line. The method can be used with a table, a graph, or any two points on the line.

Unit 2, Lesson 11

Form Follows Function

Learning Focus

Use different forms of linear and exponential functions to efficiently write equations.

Use the information given in different forms of equations to graph functions.

How do I use forms of equations for graphing linear and exponential functions?

What is the purpose of having different forms of equations?

How do I choose which form is most efficient?

Open Up the Math: Launch, Explore, Discuss

In our work so far, we have seen linear and exponential equations in many forms. Some of the forms of equations and their names are:

Linear Functions:

Equation	Name
$y = \frac{1}{2}x + 1$	Slope-intercept Form $y = mx + b$, where m is the slope and b is the y-intercept
$y = \frac{1}{2}(x - 4) + 3$	Point-slope Form $y = m(x - x_1) + y_1$, where m is the slope and (x_1, y_1) the coordinates of a point on the line
$f(0) = 1, f(n) = f(n-1) + \frac{1}{2}$	Recursive Formula $f(n) = f(n-1) + D$, Given an initial value $f(a)$ D = constant difference in consecutive terms (used only for discrete functions)

Exponential Functions:

Equation	Name
$y = 10(3)^x$	Explicit Form $y = a(b)^x$
$f(0) = 10, f(n+1) = 3f(n)$	Recursive Formula $f(n+1) = Rf(n-1)$ Given an initial value $f(a)$ R = constant ratio between consecutive terms (used only for discrete functions)

Knowing a number of different forms for writing and graphing equations is like having a mathematical toolbox. You can select the tool you need for the job, or in this case, the form of the equation that makes the job easier. Any master builder will tell you that the more tools you have the better. In this task, we'll work with our mathematical tools to be sure that we know how to use them all efficiently. As you model the situations in the following problems, think about the important information in the problem and the conclusions that can be drawn from it. Is the function linear or exponential? Does the problem give you the slope, a point, a ratio, a y-intercept? Is the function discrete or continuous? This information helps you to identify the best tools and get to work!

1. In his job as a cashier, Joe makes $\$500$ each month plus $\$20$ every time he convinces a customer to open a charge account at the store.

   a. Write the equation that describes Joe's monthly income I as a function of n, the number of accounts that Joe solicits. Name the form of the equation you wrote and why you chose to use that form.

   b. This function is:

      A. Linear B. Exponential C. Neither

   c. This function is:

      A. Continuous B. Discrete C. Neither

   d. The domain of the function is:

2.

   a. Write the equation of the line with a slope of -1 through the point $(-2,5)$.

      Name the form of the equation you wrote and why you chose to use that form.

   b. This function is:

      B. Linear B. Exponential C. Neither

   c. This function is:

      C. Continuous B. Discrete C. Neither

   d. The domain of the function is:

3.

   a. Write the equation of the geometric sequence with a constant ratio of 5 and a first term of -3. Name the form of the equation you wrote and why you chose to use that form.

   b. This function is:

      A. Linear B. Exponential C. Neither

   c. This function is:

      A. Continuous B. Discrete C. Neither

   d. The domain of the function is:

4.

   a. Write the equation of the function given on the graph. (see next page for graph)

      Name the form of the equation you wrote and why you chose to use that form.

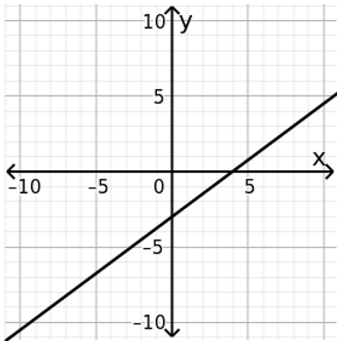
   b. This function is:

      A. Linear B. Exponential C. Neither

c. This function is:

A. Continuous B. Discrete C. Neither

d. The domain of the function is:



5. The population of the resort town of Java Hot Springs in 2013 was estimated to be $35,000$ people, with an annual rate of increase of about 2.4%.

a. Write the equation that models the number of people in Java Hot Springs, with t = the number of years from 2013.

Name the form of the equation you wrote and why you chose to use that form.

b. This function is:

A. Linear B. Exponential C. Neither

c. This function is:

A. Continuous B. Discrete C. Neither

d. The domain of the function is:

6. Yessica's science fair project involved growing some seeds to see what fertilizer made the seeds grow fastest. One idea she had was to use an energy drink to fertilize the plant. (She thought that if energy drinks make people perky, energy drinks might have the same effect on plants.) This is the data that shows the growth of the seed each week of the project.

Week	1	2	3	4	5
Height (cm)	1.7	2.9	4.1	5.3	6.5

a. Write the equation that models the growth of the plant over time.

Name the form of the equation you wrote and why you chose to use that form.

b. This function is:

A. Linear B. Exponential C. Neither

c. This function is:

A. Continuous B. Discrete C. Neither

d. The domain of the function is:

An equation gives us information that we can use to graph the function. Pick out the important information given in each of the following equations and use the information to graph the function.

7. $y = \frac{1}{2}x - 5$

8. $y = 2^n$

9. $y = -2(x + 6) + 8$

10. $f(1) = -5, f(n) = f(n - 1) + 1$

Ready for More?

Here's a challenge: Choose one of the scenarios below to write your own problem.

- Write a linear context that is continuous and would make slope-intercept the most efficient form for modeling the context.
- Write a linear table for a discrete function that would make a recursive equation the most efficient form for modeling the context.
- Write a context for a continuous exponential function that would make an explicit equation the most efficient form for modeling the context.

Takeaways

Linear Functions:

Name	What do you need to use it to write equations?	Equation
Slope-intercept Form $y = mx + b$, where m is the slope and b is the y-intercept		$y = \frac{1}{2}x + 1$
Point-slope Form $y = m(x - x_1) + y_1$, where m is the slope and (x_1, y_1) the coordinates of a point on the line		$y = \frac{1}{2}(x - 4) + 3$
Recursive Formula $f(n) = f(n - 1) + D$, Given an initial value $f(a)$ D = constant difference in consecutive terms (used only for discrete functions)		$f(0) = 1,$ $f(n) = f(n - 1) + \frac{1}{2}$

Exponential Functions:

Name	What do you need to use it to write equations?	Equation
Explicit Form $y = a(b)^x$		$y = 10(3)^x$
Recursive Formula $f(n) = Rf(n - 1)$ Given an initial value $f(a)$ R = constant ratio between consecutive terms (used only for discrete functions)		$f(0) = 10,$ $f(n) = 3f(n - 1)$

Lesson Summary

In this lesson, we summarized our work with writing equations for linear and exponential functions. We worked on strategically selecting a useful form for the context by identifying the information about the type of change and the initial values, wherever they are.