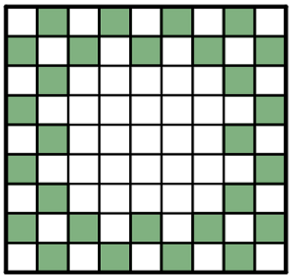
Unit 1, Lesson 1

Checkerboard Borders

Jump Start

The pattern of tiles shown here is made with a 5×5 square of white tiles surrounded by a border with alternating colored squares. Show how you would group the tiles to quickly count the number of colored squares in the pattern.



Learning Focus

Use variables to describe ways of seeing a pattern.

When I look at a pattern, how can I describe what's changing and what's staying the same?

Can seeing groups help in understanding diagrams?

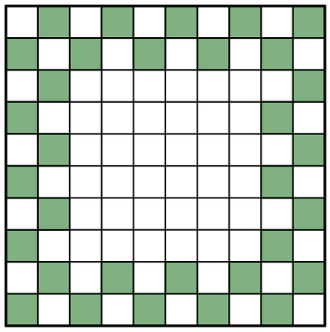
How do variables express the way we see patterns?

Open Up the Math: Launch, Explore, Discuss

In preparation for going back to school, the school administration plans to replace the tile in the cafeteria. They would like to have a checkerboard pattern of tiles two rows wide to surround the tables and serving carts.

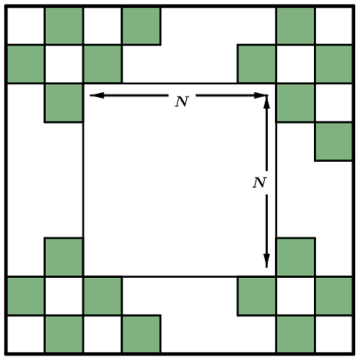
Below is an example of the border that the administration is thinking of using to surround a 6×6 square of white tiles.

1. Find the number of colored tiles in the checkerboard border using a strategy that either you or one of your peers used in the Jump Start. Show your calculations.



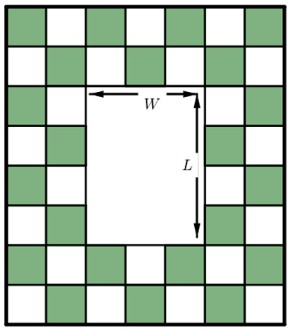
2. The contractor who was hired to lay the tile in the cafeteria is trying to generalize a way to calculate the number of colored tiles needed for a checkerboard border surrounding a square of tiles with any dimensions. To represent this general situation, the contractor started sketching the square below. Since he wants the diagram to work for any number of tiles in the middle square, he just called the side length N for the number of tiles on that side.

   Use the same counting strategy that you used for the 5×5 and the 6×6 square to find an expression for the number of colored border tiles needed for any $N \times N$ square center.



Ready for More?

Find an expression to calculate the number of colored tiles in the two-row checkerboard border for any rectangle. Be prepared to share your strategy and justify your work. Create models to assist you in your work.



Takeaways

Strategies to help us see patterns in diagrams.

Vocabulary

- equation

- **equivalent equations**
- **equivalent expressions**
- variable (algebra)

Lesson Summary

In this lesson, we found ways to see patterns in diagrams that help us to be efficient in counting. We modeled these patterns using variables to show how the pattern would work for any number of squares.

Unit 1, Lesson 2

Growing Dots

Learning Focus

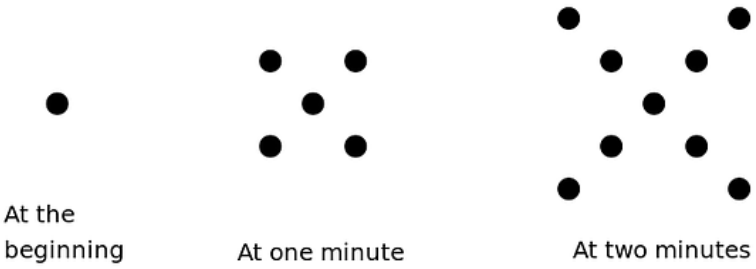
Describe a growing pattern with tables, graphs, and equations.

How does the pattern change from one figure to the next?

How can I use a mathematical representation to show how I see a pattern?

Open Up the Math: Launch, Explore, Discuss

1. Mark the way you see the pattern growing in the sequence of figures given.



2. Assuming the pattern continues in the same way, how many dots are there at 3 minutes?

3. How many dots are there at 100 minutes?

4. How many dots are there at t minutes? Model the pattern using a table, a graph, and an equation. Your model should indicate how many dots will be in the pattern at 3 minutes, 100 minutes, and t minutes. Be sure to show how your solution relates to the picture and how you arrived at your solution.

Table: Equation:	

Ready for More?

Create a pattern of dots that increases by 3 each minute, starting with $t = 0$. Write an equation that models your pattern.

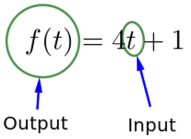
Takeaways

Representations of an arithmetic sequence:

Table: _____ Graph: _____	Explicit equation: _____ Recursive equation: _____

Adding Notation, Vocabulary, and Conventions

Arithmetic sequence: _____

Recursive thinking: _____ Recursive equation: _____ Explicit thinking: _____	Explicit equation: 

Vocabulary

- **arithmetic sequence**
- **constant difference (d) (common difference)**
- **explicit equation**
- **function notation**
- **function rule**
- **input-output pair**
- **intercepts**
- rectangular coordinate system

- **recursive equation**
- **recursive thinking**
- representations

- **sequence**
- slope

- **x-intercept**
- **y-intercept**

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we modeled a pattern using tables, graphs, equations, and diagrams. We found that this type of relationship is called an arithmetic sequence. We used recursive and explicit ways of thinking about functions, and learned to describe the relationship between inputs and outputs using function notation.

Unit 1, Lesson 3

Growing, Growing Dots

Jump Start

Which One Doesn't Belong?

A. $5 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

B. $5(2^5)$

C. $5 \cdot 128\left(\frac{1}{2}\right)^2$

D. $16(10)$

E. $20(2 \cdot 2 \cdot 2)$

Reason:

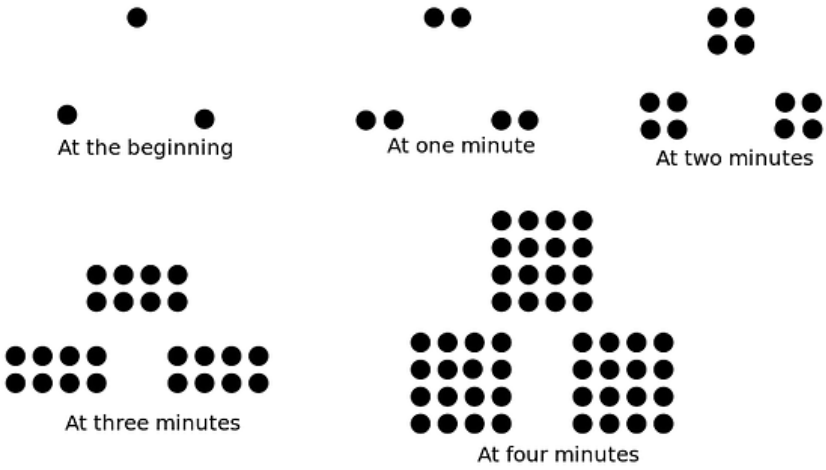
Learning Focus

Model a sequence of figures using tables, graphs, and equations.

How can I label the diagram to show how I see the pattern?

How can I use mathematical representations to show how I see the pattern changing?

Open Up the Math: Launch, Explore, Discuss



1. Describe and label the pattern of change you see in the given sequence of figures.

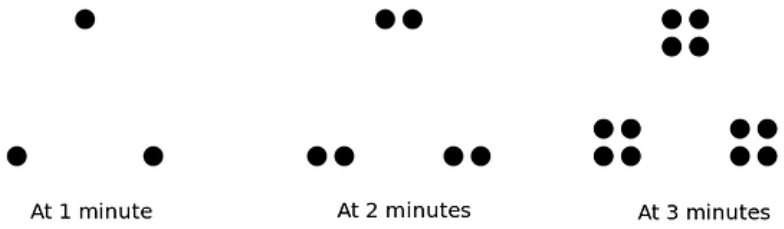
2. Assuming the sequence continues in the same way, how many dots are there at 5 minutes?

3. Model the sequence of figures using a table, graph, and recursive and explicit equations to show how many dots are in the pattern at t minutes.

Table: Recursive equation: _____ Explicit equation: _____	

Ready for More?

Write the recursive and explicit equations to model the pattern if the diagram were relabeled this way:



Takeaways

Geometric sequence: _____ Table: _____ Explicit equation: _____

Recursive equation: _____ Graph: _____

Vocabulary

- **change factor (pattern of growth)**
- **common ratio (r) (constant ratio)**
- exponent
- exponential form and expanded form
- expression
- **geometric sequence**
- prime number

Lesson Summary

In this lesson, we modeled a growing pattern using tables, graphs, and equations. We learned about a new type of function, a geometric sequence that has a common ratio between terms.

Unit 1, Lesson 4

Scott's Workout

Learning Focus

Model a sequence using a table, graph, and explicit and recursive equations.

How do I see the change between figures in each representation?

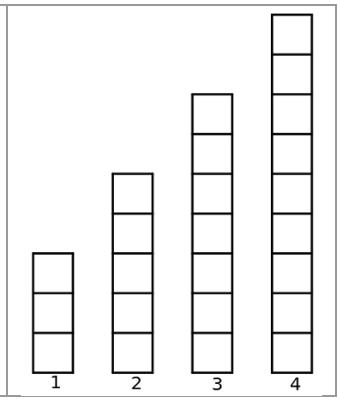
How can I tell if a sequence is arithmetic, geometric, or neither?

How are explicit equations different than recursive equations?

What are the advantages of using an explicit equation versus a recursive equation?

Open Up the Math: Launch, Explore, Discuss

Scott has decided to add push-ups to his daily exercise routine. He is keeping track of the number of push-ups he completes each day in the given bar graph, with day 1 showing he completed 3 push-ups.
After four days, Scott is certain he can continue this pattern of increasing the number of push-ups he completes each day.



1. How many push-ups will Scott do on day 10?

2. Model the number of push-ups Scott will complete on any given day. Include a table, a graph, and both recursive and explicit equations.

Table:

Recursive equation: _____
Explicit equation: _____

3. What type of sequence is your model for the number of push-ups that Scott did? How do you know?

Ready for More?

Aly is also including push-ups in her workout and says she does more push-ups than Scott because she does fifteen push-ups every day. Is she correct? Explain.

Takeaways

Finding the common difference in each representation of an arithmetic sequence:

Table: _____

Explicit equation: _____

Recursive equation: _____

Graph: _____

Adding Notation, Vocabulary, and Conventions

Recursive notation:

Vocabulary

- independent variable / dependent variable

Lesson Summary

In this lesson, we modeled a real-world situation with a table, graph, and equations, both explicit and recursive. We learned the mathematical notation for writing recursive equations and how to identify the common difference in each of the representations.

Unit 1, Lesson 5

Don't Break the Chain

Learning Focus

Represent a story context using tables, graphs, and equations.

Use function notation to write explicit and recursive equations.

How can I find the pattern of growth in a story context?

How does the pattern of growth appear in each of the representations?

Open Up the Math: Launch, Explore, Discuss

Maybe you've received an email like this before:

> Hi! My name is Bill Weights, founder of Super Scooper Ice Cream. I am offering you a gift certificate for our signature "Super Sundae" (an $\$11.95$ value) if you forward this letter to 10 people.
> When you have finished sending this letter to 10 people, a screen will come up. It will be your Super Sundae gift certificate. Print that screen out and bring it to your local Super Scooper Ice Cream store. The server will bring you the most wonderful ice cream creation in the world—a Super Sundae with three yummy ice cream flavors and three toppings!
> This is a sales promotion to get our name out to young people around the country. We believe this project can be a success, but only with your help. Thank you for your support.
> Sincerely,
> Bill Weights, Founder of Super Scooper Ice Cream

These chain emails rely on each person who receives the email to forward it on. Have you ever wondered how many people might receive the email if the chain remains unbroken? To figure this out, assume that it takes a day for the email to be opened, forwarded, and then received by the next person. On day 1, Bill Weights starts by sending the email out to his 8 closest friends. They each forward it to 10 people so that on day 2 it is received by 80 people. The chain continues unbroken.

1. How many people will receive the email on day 7?

2. How many people will receive the email on day n? Explain your answer with as many representations as possible.

Ready for More?

Although offers like Bill's are usually scams, let's think about what would happen if he really did give away all the ice cream offered in the email. If Bill gives away a Super Sundae that costs him $\$4.95$ to every person who receives the email during the first week, how much will he spend?

Takeaways

Finding the common ratio in a geometric sequence:

Table: _____

Explicit equation: _____

Recursive equation: _____

Graph: _____

Lesson Summary

In this lesson, we modeled a real-world situation with a table, graph, and equations, both explicit and recursive. We learned to use the mathematical notation for writing recursive equations for geometric sequences and how to identify the common ratio in each of the representations of a geometric sequence.

Unit 1, Lesson 6

Home on the Range

Jump Start

1. The Super Discount store is having a sale and all the clearance items are 40% off. Which of the following are correct ways to find the price of an item that is normally $\$25.00$?

   A. Multiply $\$25.00$ by 40

   B. Multiply $\$25.00$ by 0.4

   C. Multiply $\$25.00$ by 0.4 and subtract that amount from $\$25.00$

   D. Multiply $\$25.00$ by 0.6

2. What is the sale price of the item?

Learning Focus

Understand the characteristics that make sequences decrease.

Compare decreasing arithmetic and geometric sequences.

What makes an arithmetic sequence decrease?

How can a geometric sequence be decreasing?

How are equations of decreasing sequences different than increasing sequences?

Open Up the Math: Launch, Explore, Discuss

Prairie dogs are small mammals that are members of the squirrel family. They average about $14-17$ inches in size and live in underground colonies called prairie dog towns. They are native to the Great Plains region of the United States. The prairie dog population is estimated to have declined by 95% over the past 200 years, so prairie dog reserves have been established in some areas. Prairie dog reserves are places where prairie dogs can live in their natural habitat without being disturbed by hunting or other human activities.

One such prairie dog reserve was established on 120 acres of land in 2015. As the surrounding land has become more valuable for humans, the size of the reserve has been reduced over time. The following data has been collected about the size of the reserve:

Year	Size (Acres)
1	120
2	112.5
3	105
4	97.5
5	90

1. Write an explicit equation and a recursive equation to model the size of the reserve over time.

2. What type of sequence does this data represent? Justify your answer.

The sale of land at the reserve was stopped when the reserve was 76.5 acres, because biologists who study the prairie dogs found that the population was declining at the alarming rate of 20% per year. They determined that the decline was due to several factors, including the reduction of habitat from selling off the reserve land. When the reserve began, the prairie dog population on the reserve was about 600 individuals.

3. Model the prairie dog population over time, in years, starting with the beginning population. Create both an explicit equation and a recursive equation.

4. What type of sequence is represented by the prairie dog population? Justify your answer.

5. How are decreasing arithmetic sequences different from decreasing geometric sequences?

Ready for More?

What type of relationship exists between the number of acres in the reserve and the population of prairie dogs? Model the relationship with a table and a graph.

Takeaways

Decreasing arithmetic and geometric sequences:

Graph of a decreasing arithmetic sequence:	Graph of a decreasing geometric sequence:

Lesson Summary

In today's lesson, we modeled a real context with arithmetic and geometric sequences. We found that some arithmetic and geometric sequences decrease. Whether a sequence increases or decreases depends on the common difference or common ratio between terms. Additionally, we used tables, equations, and graphs to compare the behavior of decreasing arithmetic and geometric sequences.

Unit 1, Lesson 7

What Comes Next? What Comes Later?

Jump Start

1. What makes a sequence an arithmetic sequence? Provide an example.

2. What makes a sequence a geometric sequence? Provide an example.

Learning Focus

Determine if a sequence is geometric, arithmetic, or neither.

Write recursive and explicit equations for arithmetic and geometric sequences.

How can I efficiently use the information in a table to write formulas for arithmetic and geometric sequences?

Open Up the Math: Launch, Explore, Discuss

Perform each of the following steps for each table:

a. Identify if the function is arithmetic, geometric or neither. If the function is neither arithmetic nor geometric, you do not need to find any of the equations.

b. Describe how to find the next term in the sequence.

c. Write a recursive equation for the function.

d. Describe how the features identified in the recursive equation can be used to write an explicit equation for the function.

e. Write an explicit equation for the function.

Example:

x	0	1	2	3	4	…	n
y	5	8	11	14	?	…	?

a. Arithmetic, geometric, or neither? Arithmetic

b. To find the next term: add 3 to the previous term

c. Recursive equation: $f(0) = 5, f(n) = f(n-1) + 3$

d. To find the nth term: start with 5 and add 3 n times

e. Explicit equation: $f(n) = 5 + 3n$

1. Function A:

x	y
1	3
2	15
3	27
4	39
5	51
6	?
…	…
n	?

2. Function B:

x	y
1	5
2	10
3	20
4	40
5	?
…	…
n	?

3. Function C:

x	y
1	-8
2	-17
3	-26
4	-35
5	-44
6	-53
…	…
n	

4. Function D:

x	y
1	2
2	6
3	18
4	54
5	162
6	486
…	…
n	

5. Function E:

x	y
0	3
1	4
2	7
3	12
4	19
5	?
...	...
n	?

6. Function F:

x	y
1	10
2	2
3	$\frac{2}{5}$
4	$\frac{2}{25}$
5	$\frac{2}{125}$
6	$\frac{2}{625}$
...	...
n	

Ready for More?

Find the recursive and explicit equations for this function:

x	y
0	3
1	4
2	7
3	12
4	19
5	?
...	...
n	?

Takeaways

	What does it look like?	How do you use the common ratio or difference?	How do you use the first term?
Arithmetic Sequence: Recursive Equation			
Arithmetic Sequence: Explicit Equation			
Geometric Sequence: Recursive Equation			
Geometric Sequence: Explicit Equation			

Lesson Summary

In this lesson, we described the pattern of growth for arithmetic and geometric sequences and wrote recursive and explicit equations to model the sequences. We learned to identify the first term and common difference or common ratio in both the explicit and recursive forms of equations, and we developed a process for writing equations for sequences.

Unit 1, Lesson 8

What Does It Mean?

Jump Start

Find the next three terms in the sequences.

1. $7, 11, 15,$ _ , _ , _ , …
2. $-32, -39, -46,$ _ , _ , _ , …
3. $-6, 6, 18,$ _ , _ , _ , Type equation here.

Learning Focus

Find missing terms in an arithmetic sequence.

What patterns do I recognize in arithmetic sequences that can help to find missing terms?

Open Up the Math: Launch, Explore, Discuss

Each of the tables below represents an arithmetic sequence. Find the missing terms in the sequence, showing your method.

1. Complete:

x	1	2	3
y	5		11

2. Complete:

x	1	2	3	4	5
y	18				-10

3. Complete:

x	1	2	3	4	5	6	7
y	12						-6

4. Describe your method for finding the missing terms. Will the method always work? How do you know?

Here are a few more arithmetic sequences with missing terms. Complete each table, either by using the method you developed previously or by finding a new method.

5. Complete:

x	1	2	3	4
y	50			86

6. Complete:

x	1	2	3	4	5	6
y	40					10

7. Complete:

x	1	2	3	4	5	6	7	8
y	-23							5

8. Describe a method that will work to find missing terms in an arithmetic sequence, and explain why this method works.

Ready for More?

Find the 20th term for the sequence from problem 6.

x	1	2	3	4	5	6
y	40					10

Takeaways

To find missing terms in an arithmetic sequence:

Vocabulary

- **Arithmetic mean**

Bold terms are new in this lesson.

Lesson Summary

In this lesson, we found missing terms in an arithmetic sequence using several methods. We developed a formula that allows us to find the common difference for any arithmetic sequence when two terms are known. We also found an equation that can be used to find any term in an arithmetic sequence.

Unit 1, Lesson 9

Geometric Meanies

Jump Start

Which One Doesn't Belong?

Choose the number that doesn't belong with the others and explain your thinking.

(Hint: Consider the factors for the numbers.)

A. $1{,}875$

B. 405

C. $1{,}215$

D. 40

E. $1{,}280$

Reason:

Learning Focus

Apply understanding of geometric sequences to find missing terms.

How can I find missing terms in a geometric sequence?

What equations can help find missing terms in a geometric sequence?

Open Up the Math: Launch, Explore, Discuss

Each of the tables below represents a geometric sequence. Find the missing terms in the sequence, showing your method.

1. Sequence 1

    a. Complete:

x	1	2	3
y	3		12

    b. Is the missing term that you identified the only answer? Why or why not?

2. Sequence 2

    a. Complete:

x	1	2	3	4
y	7			875

    b. Are the missing terms that you identified the only answers? Why or why not?

3. Sequence 3

   a. Complete:

x	1	2	3	4	5
y	6				96

   b. Are the missing terms that you identified the only answers? Why or why not?

4. Sequence 4

   a. Complete:

x	1	2	3	4	5	6
y	4					972

   b. Are the missing terms that you identified the only answers? Why or why not?

5. Describe your method for finding the missing terms in a geometric sequence.

6. How can you tell if there will be more than one solution for the missing terms?

Ready for More?

Find the missing terms in each of the sequences:

1. Geometric:

   $4, ___, ___, ___, 324$

2. Arithmetic:

   $4, ___, ___, ___, 324$

Takeaways

Given two terms in a geometric sequence, _____

Lesson Summary

In this lesson, we found missing terms in a geometric sequence using several methods. We developed a process for finding the common ratio for any geometric sequence when two terms are known. We also found an equation that can be used to find the common ratio and any term in an arithmetic sequence.

Unit 1, Lesson 10

I Know; What Do You Know?

Learning Focus

Identify the type of sequence given any representation.

Find efficient strategies for representing sequences with tables, graphs, and equations, both explicit and recursive.

What conclusions can be drawn about a sequence, given just a few pieces of information?

Open Up the Math: Launch, Explore, Discuss

In each of the given problems, I share some of the information that I know about a sequence. Your job is to add all the things that you know about the sequence from the information that I have given. Depending on the sequence, some of the things you may be able to figure out for the sequence are:

- a table;
- a graph;
- an explicit equation;
- a recursive equation;
- the constant ratio or constant difference between consecutive terms;
- any terms that are missing;
- the type of sequence;
- a story context.

Try to find as many as you can for each sequence, but you must have at least four of the items in the list for each problem.

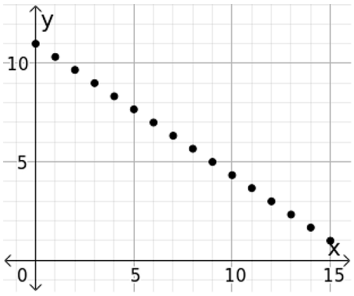
1. I know that the first five terms of the sequence are $0, -6, -12, -18, -24, \ldots$

   What do you know?

2. I know that it is a sequence where $f(1) = 5$ and the constant ratio between terms is 3.

   What do you know?

3. I know that a graph of the sequence is this:



   What do you know?

4. I know that the explicit equation for the sequence is $f(n) = -10(3)^{n-1}$.

   What do you know?

5. I know that the recursive equation for the sequence is $f(1) = -12$, $f(n) = f(n-1) + 4$.

   What do you know?

6. I know that the sequence models the value of a car that originally cost \$26,500 but loses 10% of its value each year.

   What do you know?

7. I know that the first term of the sequence is -2 and the fifth term is $-\frac{1}{8}$.

   What do you know?

Ready for More?

Work with a partner to create your own "I know; what do you know?" problems.

The options below tell you what representation to start with to create the "I know" part of the problem.

- Make an arithmetic sequence problem that starts with a graph.
- Make a geometric sequence problem that starts with a story context.
- Make an arithmetic sequence problem that starts with an explicit formula.
- Make a geometric sequence problem that starts with a recursive formula.

When you have finished with the "I know" part of the problem, trade problems with another team. You solve their problem, and they'll solve yours.

Takeaways

Given a table:	
What do you know?	What can you find?
Given a graph:	
What do you know?	What can you find?
Given an explicit equation:	
What do you know?	What can you find?
Given a recursive equation:	
What do you know?	What can you find?

Lesson Summary

In this lesson, we found efficient strategies for identifying and representing arithmetic and geometric sequences, whatever information is given. We sharpened our skills in finding the first term and common difference or common ratio in any representation and how to use one representation to find another.